

# Attitude Estimation of Four-Rotor UAV Based on Extended Kalman Filter

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**Abstract:** Quadrotor unmanned aerial vehicle (UAV) is a typical multi-input multi-output (MIMO), nonlinear and strong coupling underactuated system. In the working process of the system, it is necessary to perform information fusion on the attitude detected by the sensor to achieve accurate measurement of attitude angle and angular velocity. Accurate and efficient measurement of UAV attitude angle is the basis of UAV flight control. In this paper, the extended Kalman filter (EKF) algorithm is used to estimate the attitude information of the four-rotor UAV. Firstly, a four-rotor UAV simulation model is established on Simulink in Matlab, and then the attitude information of the UAV is measured and estimated. The results show that the extended Kalman filter algorithm can effectively estimate the attitude information of UAV.

**Keywords:** Four-Rotor UAV, Extended Kalman Filter, Matlab Simulation, Attitude Estimation.

## Introduction

In recent years, multi-rotor UAV has become a research hotspot, the results are widely used in meteorological monitoring, low-altitude reconnaissance, light transport and other fields ([Paw & Balas, 2011](#)). Multi-rotor UAV usually has more than two rotor propellers, with vertical take-off and landing, simple structure, high mobility, good safety, easy maintenance. It is an unstable strongly coupled system with 6 degrees of freedom and 4 inputs. Therefore, it is very useful to improve the stability and controllability of UAV to make it autonomous flight or manual guidance. Attitude measurement is an important part of UAV control system. The accuracy of attitude measurement system directly affects the control performance. Therefore, it is necessary to establish UAV attitude measurement system ([T. Zhang & Liao, 2017](#)). In recent years, UAV attitude estimation has been studied more mature and widely used algorithms are based on Kalman filter (KF) ([Hajiyev & Soken, 2013](#); [OUYANG & Zhou, 2014](#); [Wu et al., 2011](#)), complementary filtering ([Du et al., 2015](#); [Mahony et al., 2008](#)), gradient

descent method ([Peng et al., 2015](#)) of the Euler Angle method ([Zhi-ju et al., 2010](#)), quaternion method ([Chen et al., 2015](#); [LI et al., 2006](#)), direction cosine matrix method and equivalent rotation vector method ([Z. Zhang & Duan, 2010](#)). The complementary filtering and gradient descent algorithm are simple and suitable for processing aircraft with limited performance. Under the condition of hardware performance, KF is mostly used KF is effective and widely used, but not suitable for nonlinear systems including UAV flight control system. In view of this limitation, there are many extended algorithms based on KF, such as unscented Kalman filter (UKF) ([Julier et al., 2000](#)), particle filter (PF), extended Kalman filter (EKF). UKF algorithm and PF algorithm have the problems of large amount of calculation and poor real-time performance, while EKF has small data storage and relatively small amount of calculation. In summary, EKF is the best choice for data fusion and attitude estimation of four-rotor UAV.

Extended Kalman Filter (EKF) was introduced to integrate attitude data from different sources, but only simulation results were given. The extended Kalman filter method for attitude estimation was analyzed in ([Huang et al., 2005](#)) The Gauss-Newton iteration method was used to calculate the accelerometer vector and the magnetometer vector as the measurement vectors, and the measurement data collected from the sensor was used to test the filter. In ([Xue et al., 2009](#)), an extended Kalman filter algorithm based on quaternion is introduced to improve the accuracy of attitude estimation. The algorithm uses an improved Gauss-Newton algorithm. Firstly, the attitude model and attitude dynamics model of UAV are established. Then the basic principle of extended Kalman filter algorithm is expounded. Finally, the simulation model of UAV is established in Simulink of Matlab, and the effectiveness of extended Kalman filter algorithm is verified. The results show that the attitude solver designed by EKF algorithm can provide reliable attitude feedback for UAV flight control, improve the accuracy of attitude measurement, and meet the needs of UAV attitude control.

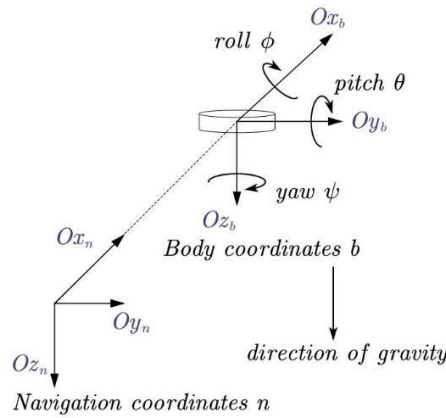
## Research Method

### UAV Attitude Modeling

#### Attitude Model

When studying the flight state of the rotor UAV, the navigation coordinate system ( $n$  system) and the body coordinate system ( $b$  system) are mainly used. The navigation coordinate system is the northeast coordinate system. The body coordinate system is fixed on the origin of the rotorcraft UAV,  $Ox_b$  points forward,  $Oy_b$  points to the right,  $Oz_b$  points down. The roll angle

$\phi$  of the UAV corresponds to the rotation around the  $Ox_b$  axis, the pitch angle  $\theta$  corresponds to the rotation of the  $Oy_b$  axis, and the yaw angle  $\psi$  corresponds to the rotation of the  $Oz_b$  axis. The navigation coordinate system and the body coordinate system of the rotor UAV are shown in Figure 1.



**Figure 1 Navigation Coordinate System and Fuselage Coordinate System of Rotor UAV**

Euler Angle, direction cosine and quaternion are commonly used positioning methods in attitude measurement system. In attitude estimation, the measurement vector of body coordinate is transformed into navigation coordinate by coordinate transformation matrix  $C_b^n$ . In Euler angles, the representation of the transformation matrix  $C_b^n$  is defined as Equation 1 :

$$C_b^n = \begin{bmatrix} \cos \phi \cos \psi + \sin \phi \sin \theta \sin \psi & \cos \theta \sin \psi & \sin \phi \cos \psi - \cos \phi \sin \theta \sin \psi \\ -\cos \phi \sin \psi + \sin \phi \sin \theta \cos \psi & \cos \theta \cos \psi & -\sin \phi \sin \psi - \cos \phi \sin \theta \cos \psi \\ -\sin \phi \cos \theta & \sin \theta & \cos \phi \cos \theta \end{bmatrix} \quad (1)$$

Quaternion attitude estimation can reduce the amount of calculation and avoid Euler angle singularity problem. The expression of quaternion is  $q = [q_0 \ q_1 \ q_2 \ q_3]^T$ . In quaternion, the body coordinates can be obtained by continuous rotation transformation of navigation coordinates. The inverse matrix of the transformation matrix is its own transpose. The quaternion coordinate transformation matrix from navigation coordinates to body coordinates is Equation 2 :

$$C_n^b(\vec{q}) = \begin{bmatrix} q_0^2 + q_1^2 - q_2^2 - q_3^2 & 2(q_1q_2 + q_0q_3) & 2(q_1q_3 - q_0q_2) \\ 2(q_1q_2 - q_0q_3) & q_0^2 - q_1^2 + q_2^2 - q_3^2 & 2(q_2q_3 + q_0q_1) \\ 2(q_1q_3 + q_0q_2) & 2(q_2q_3 - q_0q_1) & q_0^2 - q_1^2 - q_2^2 + q_3^2 \end{bmatrix} \quad (2)$$

Combined with Equation 1 and Equation 2, the attitude angle is obtained as follows Eq 3 :

$$\begin{cases} \theta = \arcsin [2(q_2q_3 - q_0q_1)], [-\frac{\pi}{2}, \frac{\pi}{2}] \\ \phi = -\arctan \left[ \frac{2(q_1q_3 + q_0q_2)}{q_0^2 - q_1^2 - q_2^2 + q_3^2} \right], [-\pi, \pi] \\ \psi = \arctan \left[ \frac{2(q_1q_2 - q_0q_3)}{q_0^2 - q_1^2 + q_2^2 - q_3^2} \right], [-\pi, \pi] \end{cases} \quad (3)$$

In strap down inertial navigation system, the relationship between angular velocity and unit quaternion is as follows Equation 4:

$$\begin{bmatrix} \dot{q}_0 \\ \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 0 & -\omega_x & -\omega_y & -\omega_z \\ \omega_x & 0 & \omega_z & -\omega_y \\ \omega_y & -\omega_z & 0 & \omega_x \\ \omega_z & \omega_y & -\omega_x & 0 \end{bmatrix} \begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix} \quad (4)$$

Where  $\omega_x, \omega_y, \omega_z$  is the real rotation rate around  $x, y, z$  axis in the body coordinate measured by the gyroscope.

### Model of Attitude Dynamics

The attitude dynamics model of the four-rotor UAV is shown in Figure 2.

From Euler equation 5 :

$$J\dot{\omega}^b + \omega^b \times J\omega^b = G_a + \tau \quad (5)$$

Where  $\omega^b$  represents the angular velocity in the body coordinate, and  $p, q, r$  is used to represent the three components of  $\omega^b$  on the body axis:  $\omega_x, \omega_y, \omega_z$ , that is,  $[p \ q \ r] = [\omega_{xb} \ \omega_{yb} \ \omega_{zb}]$ ;  $\tau$  represents the torque generated by the propeller on the body axis, including the rolling torque  $\tau_x$  around the  $Ox_b$  axis, the pitching torque  $\tau_y$  around the  $Oy_b$  axis, and the yaw torque  $\tau_z$  around the  $Oz_b$  axis.

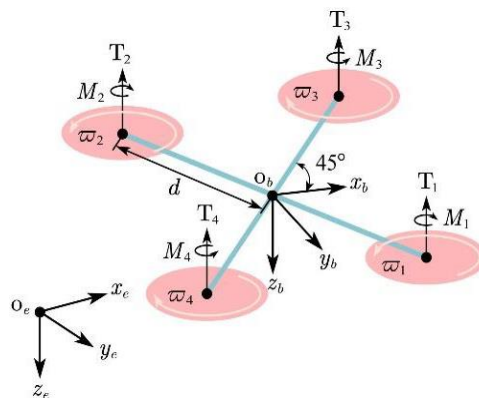


Figure 2 Attitude Dynamics Model of Four-Rotor UAV

$G_a$  represents the gyro moment, which is equivalent to a gyro when the motor is rotating at high speed. High speed rotating gyro is a very stable individual, with the ability to maintain its own axial unchanged. Therefore, if there is an external force to change the direction of the gyro shaft, then a gyro torque will be generated to resist this change. The calculation formula of  $G_a$  is as follows Equation 6:

$$G_a = \begin{bmatrix} G_{a,\phi} \\ G_{a,\theta} \\ G_{a,\psi} \end{bmatrix} = \begin{bmatrix} qJ_{RP}(\bar{\omega}_1 + \bar{\omega}_2 - \bar{\omega}_3 - \bar{\omega}_4) \\ pJ_{RP}(-\bar{\omega}_1 - \bar{\omega}_2 + \bar{\omega}_3 + \bar{\omega}_4) \\ 0 \end{bmatrix} \quad (6)$$

Where  $J_{RP}$  represents the total rotational inertia of the whole motor rotor and propeller around the body shaft;  $\bar{\omega}_1, \bar{\omega}_2, \bar{\omega}_3, \bar{\omega}_4$  denotes the rotational speed of propellers 1,2,3,4.

Assuming that the four-rotor aircraft is a uniform and symmetrical rigid body, the mass and moment of inertia of the four-rotor aircraft do not change. The inertia matrix  $J$  can be expressed as Equation 7:

$$J = \begin{bmatrix} J_{xx} & 0 & 0 \\ 0 & J_{yy} & 0 \\ 0 & 0 & J_{zz} \end{bmatrix} \quad (7)$$

Substituting Equation 7 into Equation.5, we can get Eq.8 and Eq.9 :

$$\begin{bmatrix} J_{xx}\dot{p} \\ J_{yy}\dot{q} \\ J_{zz}\dot{r} \end{bmatrix} = \begin{bmatrix} qr(J_{yy} - J_{zz}) \\ pr(J_{zz} - J_{xx}) \\ pq(J_{xx} - J_{yy}) \end{bmatrix} + \begin{bmatrix} -qJ_{RP}\Omega \\ pJ_{RP}\Omega \\ 0 \end{bmatrix} + \begin{bmatrix} \tau_x \\ \tau_y \\ \tau_z \end{bmatrix} \quad (8)$$

$$\begin{cases} \dot{p} = \frac{1}{J_{xx}} [\tau_x + qr(J_{yy} - J_{zz}) - qJ_{RP}\Omega] \\ \dot{q} = \frac{1}{J_{yy}} [\tau_y + pr(J_{zz} - J_{xx}) + pJ_{RP}\Omega] \\ \dot{r} = \frac{1}{J_{zz}} [\tau_z + pq(J_{xx} - J_{yy})] \end{cases} \quad (9)$$

Where  $\Omega = -\bar{\omega}_1 - \bar{\omega}_2 + \bar{\omega}_3 + \bar{\omega}_4$ .

## Extended Kalman Filter (EKF)

The state equation of the extended Kalman filter uses the non-linear stochastic difference equation to estimate the current system state using the previous system state:

$$\mathbf{x}_k = f(\mathbf{x}_{k-1}, \mathbf{u}_{k-1}, \mathbf{w}_{k-1}) \quad (10)$$

Ignore the  $u$  control input, we can get:

$$\mathbf{x}_k = f(\mathbf{x}_{k-1}, 0, \mathbf{w}_{k-1}) \quad (11)$$

The measurement equation for the current state is:

$$\mathbf{z}_k = h(\mathbf{x}_k, \mathbf{v}_k) \quad (12)$$

Where  $k - 1$  and  $k$  represent the previous state and the current state respectively;  $\mathbf{x} \in R^n$  denotes the state to be estimated;  $\mathbf{u} \in R^1$  represents optional input control, which is generally ignored in practice;  $\mathbf{z} \in R^m$  represents the measured value;  $\mathbf{w} \in R^n$  represents process noise, when from the previous state into the current state, there will be many external factors interference;  $\mathbf{v} \in R^m$  represents the measurement noise, mainly any measuring instrument will have a certain error;  $f$  denotes the nonlinear mapping equation from the previous state to the current state;  $h$  denotes the nonlinear mapping equation between state and measurement. Assume that the process noise  $w$  and the measurement noise  $v$  are independent of each other and follow a Gaussian distribution:

$$\begin{aligned} p(w) &\sim N(0, Q) \\ p(v) &\sim N(0, R) \\ Q &= E[\mathbf{w}\mathbf{w}^T] \\ R &= E[\mathbf{v}\mathbf{v}^T] \end{aligned}$$

Where  $Q^{n \times n}$  represents the covariance matrix of process noise  $\mathbf{w}$ , represents the correlation between  $\mathbf{w}$  vector elements;  $R^{m \times m}$  represents the covariance matrix of the measurement noise  $\mathbf{v}$ , representing the correlation between the  $\mathbf{v}$  vector elements;  $Q, R$  changes with state.

Based on the above definition, ignoring the process noise  $\mathbf{w}$  and the control input  $\mathbf{u}$ , it can get:

$$\bar{\hat{\mathbf{x}}}_k = f(\hat{\mathbf{x}}_{k-1}, 0, 0) \quad (13)$$

Similarly, ignoring the measurement noise  $\mathbf{v}$ , we can obtain:

$$\bar{\mathbf{z}}_k = h(\bar{\hat{\mathbf{x}}}_k, 0) \quad (14)$$

where,  $\bar{\hat{\mathbf{x}}}_k$  represents the prior state estimation of the current state obtained only by using the prior knowledge of the process without considering the process noise;  $\hat{\mathbf{x}}_k$  represents the posterior state estimation of the current state obtained by using the measured value  $\mathbf{z}_k$ , which is the final state.  $\bar{\mathbf{z}}_k$  represents the noise-free measurement value obtained by ignoring the measurement noise according to the current prior state.

Rewrite the above model to get:

$$\begin{aligned}
 \mathbf{x}_k &\approx \bar{\hat{x}}_k + A(\mathbf{x}_{k-1} - \hat{\mathbf{x}}_{k-1}) + W\mathbf{w}_{k-1} \\
 \mathbf{z}_k &\approx \bar{\mathbf{z}}_k + H(\mathbf{x}_k - \bar{x}_k) + V\mathbf{v}_k \\
 p(W\mathbf{w}_{k-1}) &\sim N(0, WQW^T) \\
 p(V\mathbf{v}_k) &\sim N(0, VRV^T)
 \end{aligned}
 \tag{15}$$

Here,  $\mathbf{x}_k$  and  $\mathbf{z}_k$  represent the real state vector and the measurement vector respectively;  $\bar{\hat{x}}_k$  and  $\bar{\mathbf{z}}_k$  represent the state vector and measurement vector without noise;  $\mathbf{w}_k$  and  $\mathbf{v}_k$  represent process noise and measurement noise, respectively.

$A$  denotes the Jacobian matrix of the partial derivative of function  $f$  with respect to  $\mathbf{x}$ :

$$A = \left[ \begin{array}{cccc} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \dots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \dots & \frac{\partial f_2}{\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \dots & \frac{\partial f_n}{\partial x_n} \end{array} \right]_{(\hat{x}_{k-1}, 0, 0)}
 \tag{16}$$

$W$  denotes the Jacobian matrix of the partial derivative of function  $f$  with respect to  $\mathbf{w}$ :

$$W = \left[ \begin{array}{cccc} \frac{\partial f_1}{\partial w_1} & \frac{\partial f_1}{\partial w_2} & \dots & \frac{\partial f_1}{\partial w_n} \\ \frac{\partial f_2}{\partial w_1} & \frac{\partial f_2}{\partial w_2} & \dots & \frac{\partial f_2}{\partial w_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial w_1} & \frac{\partial f_n}{\partial w_2} & \dots & \frac{\partial f_n}{\partial w_n} \end{array} \right]_{(\hat{x}_{k-1}, 0, 0)}
 \tag{17}$$

$H$  denotes the Jacobian matrix of the partial derivative of function  $h$  with respect to  $\mathbf{x}$ :

$$H = \left[ \begin{array}{cccc} \frac{\partial h_1}{\partial x_1} & \frac{\partial h_1}{\partial x_2} & \dots & \frac{\partial h_1}{\partial x_n} \\ \frac{\partial h_2}{\partial x_1} & \frac{\partial h_2}{\partial x_2} & \dots & \frac{\partial h_2}{\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial h_n}{\partial x_1} & \frac{\partial h_n}{\partial x_2} & \dots & \frac{\partial h_n}{\partial x_n} \end{array} \right]_{(\hat{x}_k, 0, 0)}
 \tag{18}$$

$V$  denotes the Jacobian matrix of the partial derivative of function  $h$  with respect to  $\mathbf{v}$ :

$$V = \begin{bmatrix} \frac{\partial h_1}{\partial v_1} & \frac{\partial h_1}{\partial v_2} & \dots & \frac{\partial h_1}{\partial v_n} \\ \frac{\partial h_2}{\partial v_1} & \frac{\partial h_2}{\partial v_2} & \dots & \frac{\partial h_2}{\partial v_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial h_n}{\partial v_1} & \frac{\partial h_n}{\partial v_2} & \dots & \frac{\partial h_n}{\partial v_n} \end{bmatrix}_{(\hat{x}_k, 0, 0)} \quad (19)$$

Because  $\mathbf{x}$  changes with  $k$ , the above Jacobian matrix  $A, W, H, V$  also changes with  $k$ , which is different from the linear Kalman filter algorithm.

A prior estimation error is defined according to the above formula:

$$\bar{e}_{x_k} \approx A(\mathbf{x}_{k-1} - \hat{\mathbf{x}}_{k-1}) + W\mathbf{w}_{k-1} \quad (20)$$

$$\bar{e}_{z_k} \approx H(\mathbf{x}_k - \hat{\mathbf{x}}_k) + V\mathbf{v}_k \approx H\mathbf{e}_{x_k} + V\mathbf{v}_k \quad (21)$$

$$p(\bar{e}_{x_k}) \sim N(0, \bar{P}_k) \quad (22)$$

$$\bar{P}_k = E[\bar{e}_{x_k} \bar{e}_{x_k}^T] = A P_{k-1} A^T + W Q W^T \quad (23)$$

$$P_k = E[\mathbf{e}_{x_k} \mathbf{e}_{x_k}^T] = (I - K_k H) \bar{P}_k \quad (24)$$

$$K_k = \frac{\bar{P}_k H^T}{H \bar{P}_k H^T + V R V^T} \quad (25)$$

where  $\bar{P}_k$  and  $P_k$  represent the covariance matrix of the prior estimation error and the posterior estimation error, respectively, and  $K_k \in R^{n \times m}$  represents the Kalman gain or the equalization coefficient.

Suppose that the result of the error estimate is  $\hat{e}_k$ , then the posterior state estimate can be obtained:

$$\mathbf{x}_k = \bar{\mathbf{x}}_k + \hat{e}_k \quad (26)$$

Set the initial  $\bar{e}_k = 0$ , using the basic Kalman filter method to derive:

$$\hat{e}_k = K_k \bar{e}_{z_k} \quad (27)$$

From this, the final state estimation is:

$$\hat{\mathbf{x}}_k = \bar{\mathbf{x}}_k + K_k (\mathbf{z}_k - h(\bar{\mathbf{x}}_k, 0)) \quad (28)$$



According to the Equation 13 and Equation 23, the prior state is estimated, and then the posterior state is estimated according to the Equation 24, Equation 25 and Equation 28, so as to estimate the current state iteratively and recursively.

## Result and Discussion

### Simulation Platform Establishment

According to the above established UAV attitude model and the mathematical model of extended Kalman filter, the validity of extended Kalman filter is verified on Matlab, and the simulation model of four-rotor UAV is established on Simulink. The simulation platform of four-rotor UAV and the model of four-rotor UAV are shown in Figure 3 and Figure 4.

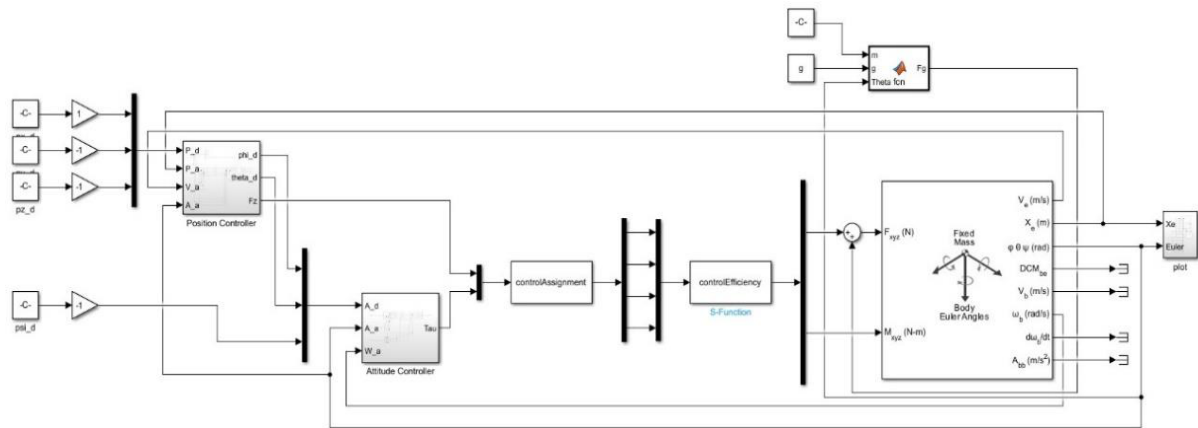


Figure 3 Four-Rotor UAV Simulation Model

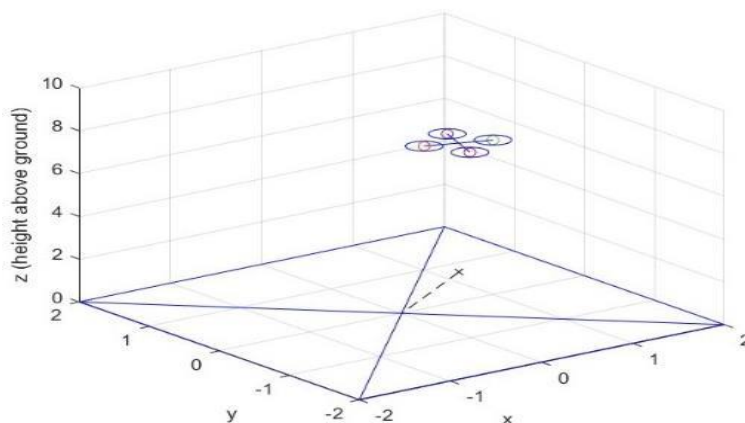


Figure 4 Four-Rotor UAV Model

The four-rotor UAV controls the flight state of the UAV by controlling the speed of the four motors, such as position, speed, attitude and angular velocity. The four-rotor UAV is a typical multi-input multi-output (MIMO), nonlinear and strongly coupled underactuated system. When designing the control rate, it is necessary to combine the dynamic model of the UAV and adopt the structure of inner and outer loop control. The inner loop is used to control the

attitude, and the outer loop is used to control the position. These two controllers have their own inner and outer loops to control the angular velocity, attitude and linear velocity and position respectively. The four-rotor UAV simulation model in Figure 3 includes UAV position controller, attitude controller, control allocation module, control efficiency module, dynamics module, gravity module and visualization module.

The position controller is used to control the position of the UAV. The inner loop uses the PID controller to control the line speed, and the outer loop uses the P controller to control the position. Similarly, the attitude controller is used to control the attitude of the UAV. The inner loop uses the PID controller to control the angular velocity, and the outer loop uses the P controller to control the angle. The control distribution module converts the expected force and torque into the expected speed of the motor; control efficiency module, which calculates the force and torque according to the speed of the motor. The dynamics module is used to represent the UAV body, and calculate the position, velocity, attitude, angular velocity and other information of the UAV according to the input force and torque. a gravity module for applying gravity to the UAV; finally, the visualization module, using the robot toolbox for visual display, as shown in Figure 4

## Simulation Result

The real state information of the UAV in the above UAV simulation model is obtained, and the Gaussian white noise is added. The mean value is 0 and the variance is 1. The initial state is  $[\theta \ \dot{\theta} \ \phi \ \dot{\phi} \ \psi \ \dot{\psi}] = [1 \ 2 \ 3 \ 3 \ 2 \ 2]$ , the sampling time is set to 0.1 s, and the state estimation of the UAV is shown in Figure 5, Figure 6 and Figure 7.

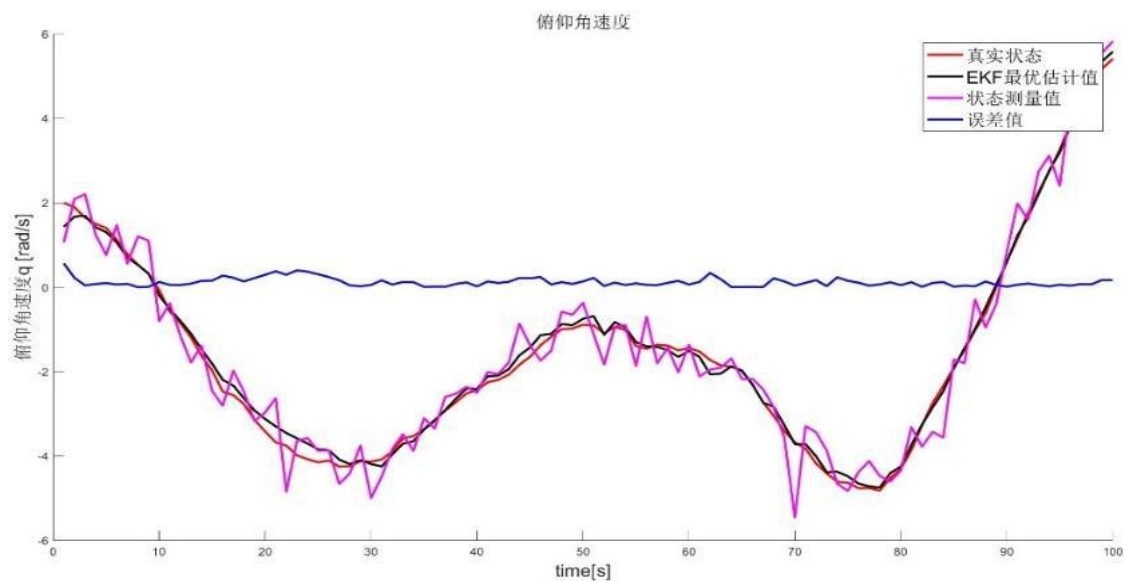


Figure 5 Estimation of Pitch Angular Velocity  $\dot{\theta}$

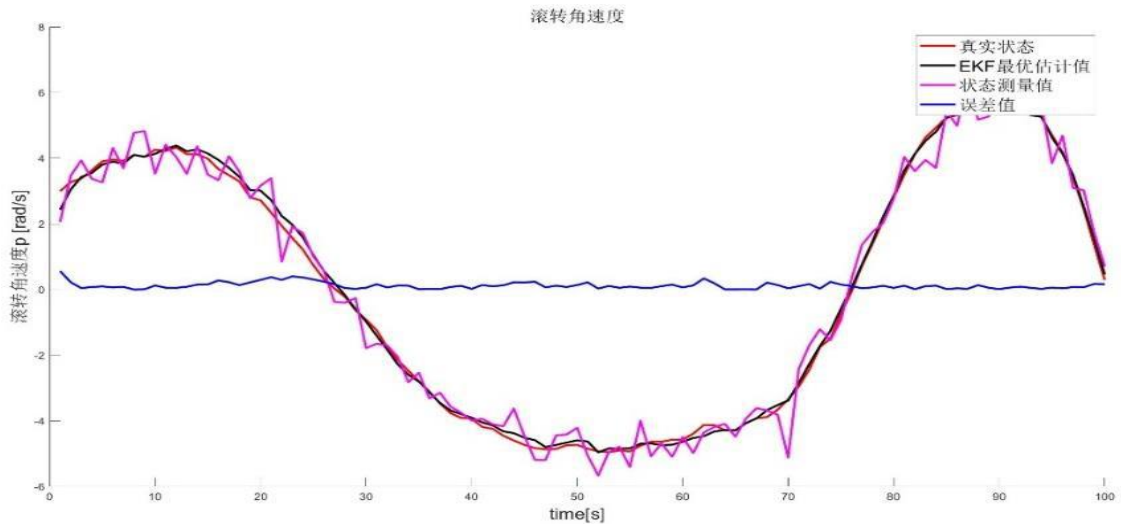


Figure 6 Estimation of Roll Angular Velocity  $\phi$

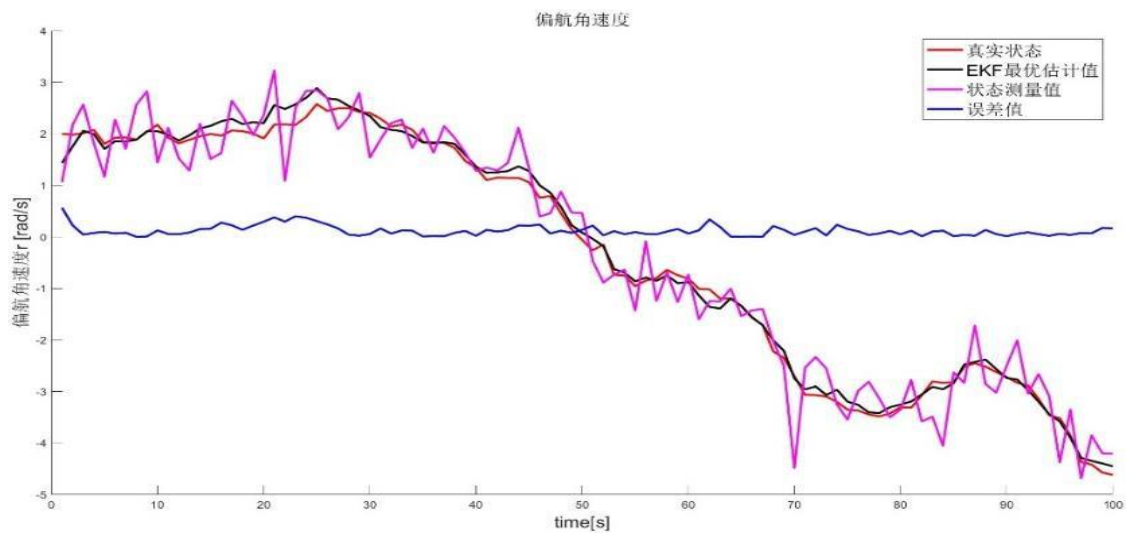


Figure 7 Estimation of Yaw Angular Velocity  $\psi$

Figure 5, Figure 6 and Figure.7 respectively represent the estimation of pitch angular velocity, roll angular velocity and yaw angular velocity. It can be seen from the information in the figure that there is a certain noise in the measured value of the state. The extended Kalman filter algorithm can effectively estimate the real state of the UAV. The absolute value of the error is kept within  $0 \sim 0.5$ , which proves the effectiveness of the extended Kalman filter.

## Conclusions

In this paper, the extended Kalman filter (EKF) algorithm is used to estimate the attitude information of the four-rotor UAV. Firstly, a four-rotor UAV simulation model is established on Simulink in Matlab, and then the attitude information of the UAV is measured and estimated. The results show that the extended Kalman filter algorithm can effectively estimate

the attitude information of UAV. The simulation results show that the attitude solver designed by EKF algorithm can provide reliable attitude feedback for UAV flight control, improve the accuracy of attitude measurement, and meet the needs of UAV attitude control.

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