The Feasibility of Trajectory Controller by SS Equation of an Error Model in WMR

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Abstract: There has been an increasing amount of research on WMRs. The WMRs have lots of advantages: their mechanical system is usually simple; the whole movement mostly contains two dependent motors; and their motions in two-dimensional space are quite adaptable. At the same time, the WMRs have better power efficiency. WMRs have been studied from many points of view, such as modelling control, obstacle avoidance, localization, tip-over stability, and so on. This paper discusses the control algorithm. WMRs have no holonomic constraints. That is, the mobile robot may not be able to reach a specific position and angle in a specific time; however, the robot can reach the target in a different motion or within a limited time. In this paper, an error model based on the WMR is introduced. The model explains the error between the actual position and the ideal or target position in the continuous motion of WMRs. The error model will be transformed into a state-space equation in order to demonstrate that the algorithm that controls the WMR`s movement is controllable. By splitting the inputs u into u_F and u_B, the whole equation is transformed into a zero-input state-space system. Then, the errors can be minimized with the optimization of the feedback. The results show that by optimizing the time-varying feedback matrix, the error is asymptotically stable and close to zero. Therefore, by adding this control law to the controller, an approach to making mobile robots follow the planned trajectory is obtained.

Keywords: WMR; Tractor Control Algorithm, Error-Model, State-Space Equation

Introduction

The WMRs are mobile robots driven by wheel. In recent years there has been an increasing amount of research on the WMRs. As one of the easiest mobile robots, the WMRs have lots of advantages. First, their mechanical system is usually simple. The whole movement only contains two dependent motors mostly (Suwoyo & Harris Kristanto, 2022). Also, their

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motions in the 2 dimensions' space are rather flexible. At the same time, the WMRs have better power efficiency. Thus, mobile robots are increasingly used in industry, in service robotics, for domestic needs, in difficult-to-access or dangerous areas. Mobile robots have been studied in many points of view such as modelling control, obstacle a voidance, localization, tip-over stability and so on. And, in this paper, we will discuss the control algorithm. It should be point out that the WMRs have nonholonomic constraints (Geng et al., 2022). That means the mobile robot may not have the ability to reach some exact position and angle in certain time, however, the robots can reach the target in another motion or in some limited times. Several controllers were proposed for mobile robots with nonholonomic constraints, where the two main approaches to controlling mobile robots are posture stabilization and trajectory tracking (Nwachioma & Pérez-Cruz, 2021).

The aim of posture stabilization is to stabilize the robot to a reference point, while the aim of trajectory tracking is to have the robot follow a reference trajectory. For mobile robot trajectory tracking is easier to achieve than posture stabilization. This comes from the assumption that the wheel makes perfect contact with the ground, meaning that the robots will have no slipping (Houtman et al., 2021). Trajectory tracking is more natural for mobile robots. Usually, the reference trajectory is obtained by using a reference robot. Therefore, all the kinematic constraints are implicitly considered by the reference trajectory (Luo et al., 2023). The control inputs are mostly obtained by a combination of feedforward inputs, calculated from reference trajectory, and feedback control law. (Lim et al., 2023; Xue et al., 2021) The stabilization to reference trajectory requires a nonzero motion condition. Predictive control techniques are a very important area of research (Belousov et al., 2022). In the field of mobile robotics predictive approached to path tracking also seem to be very promising because the reference trajectory is known beforehand. Most mobile-based predictive controllers use a linear model of mobile robot kinematics to predict future system output. The reminder of the paper is organized as follows. The section 2 is a description of the mobile robot, its control architecture and its kinematics. The concept of trajectory tracking design and error-model are introduced in section 3. In section 4, there will be a conclusion, and the reference will be given in final (Zhao et al., 2023).

Research Method

Mobile Robot Control-System Design

In this paper, a small, two-wheeled differentially driven mobile robot will be showed, as Figure 1.

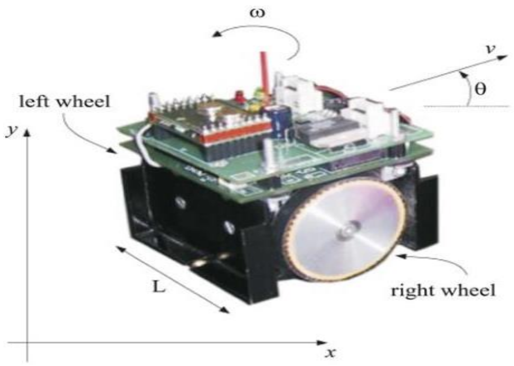

Figure 1 Description of the WMR

Robot Description and Control Architecture

The model of the robot, as has mentioned lots of before, contains 2 differentially motors. To balance the robot, there`s also an idle wheel situated in the robot (Mahapatro et al., 2023). The control of the mobile robot`s motion is performed on two levels, as demonstrated in Figure 2. The low-level control is in charge of controlling the robot`s wheel speeds, while the high-level control determines the required robot speeds considering its first-order kinematics (Koukas et al., 2022; Luo et al., 2023; Teck & Dewil, 2022; Zhao et al., 2023). This two-layer architecture is very common in practice because most mobile robots and manipulators usually do not allow the user to impose accelerations or torques at the inputs. It can also be viewed as a simplification to the problem as well as a more modular design approach. In this paper, the low-level control is implemented in the robot with basic PID controllers. We assume that the wheel can get its speed as quickly as we set (Liu et al., 2023).

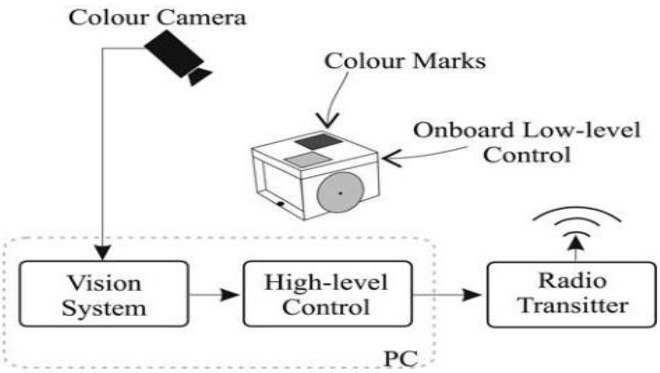

Figure 2 System Overview

Kinematics

The robot`s architecture, together with its symbols, is shown in Figure 1. The kinematic motion equations of the mobile robot are equivalent to those for a unicycle (Belousov et al., 2022). Robots with such an architecture have a nonintegrable constrain in the form Equation 1:

$$A(q)\dot{q} = \dot{x}sin\theta - \dot{y}cos\theta \quad (1)$$

Resulting from the assumption that the robot cannot slip in a lateral direction. In Equation 1, $A(q)$ is the constraint matrix defined over the generalized coordinates. $q(t) = [x(t)\ y(t)\ \theta(t)]^T$. The first-order kinematics model is obtained by expressing all the achievable velocities of the mobile robot as a linear combination of the vector fields $s_i(q)$ that span the null space of the matrix $A(q)$. The kinematics model then results in the following equation 2:

$$\dot{q}(t) = [s_1(q)\ \ s_2(q)]\begin{bmatrix} v(t) \\ w(t) \end{bmatrix} = \begin{bmatrix} cos\theta(t) & 0 \\ sin\theta(t) & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} v(t) \\ w(t) \end{bmatrix} \quad (2)$$

where $v(t)$ and $w(t)$ are the tangential and angular velocities of the platform in Figure 1. The right and left velocities of the robot`s wheels (needed for the low-level control) are then expressed as $v_R = v + \frac{wL}{2}$ and $v_L = v - \frac{wL}{2}$.

Driving Constraints

During low-level control the bounded velocity and acceleration constraints are considered. The robot`s tangential and angular velocities are bounded with $v_{MAX} = w_M R$ and $w_{MAX} = 2w_M/L$, where w_M is the maximum angular velocity of the wheel and R is its radius. A saturation of the command velocities that preserve the current curvature $k = \frac{w}{v}$ is performed as:

$$\sigma = max\{|v|/v_{MAX}, |w|/w_{MAX}\}$$

Thus, the actual command velocities v_c and w_c stand for

$$v_c = sign(v) \cdot v_{MAX}, w_c = \frac{w}{\sigma}; \sigma = \frac{|v|}{v_{MAX}}$$

$$v_c = \frac{v}{\sigma}, w_c = sign(w) \cdot w_{MAX}; \sigma = \frac{|w|}{w_{MAX}}$$

$$v_c = v, w_c = w;\ \sigma = 1$$

Note that in this method, w_M is not physical limit of the wheels, like the motor`s max speed. In fact, it`s a limit defined by designer to get the ability of preserving the curvature. During the acceleration of robot, it`s also necessary to consider the slipping over the wheels. To speed up the robot, there`s a torque set on the wheel which produces the acceleration. Therefore, we need to situate a limit of the acceleration to prevent the slipping. So, the wheel`s command velocities (v_{Rc}, v_{Lc}) are bounded with the allowable acceleration as follows:

$$\begin{aligned} v_{Rc}(k) &= v_R(k); & |a_R| \leq a_{MAX} \\ v_{Rc}(k) &= v_{Rc}(k-1) + sign(a_R) \cdot a_{MAX} \cdot T_s; & |a_R| \leq a_{MAX} \end{aligned}$$

and the same to v_L:

$$\begin{aligned} v_{Lc}(k) &= v_L(k); & |a_L| \leq a_{MAX} \\ v_{Lc}(k) &= v_{Lc}(k-1) + sign(a_L) \cdot a_{MAX} \cdot T_s; & |a_L| \leq a_{MAX} \end{aligned}$$

The a_{MAX} above is usually obtained by experiments, considering the different texture of the ground in working circumstance.

Result and Discussion

Definition of the Trajectory-Tracking Problem and the Error-Model

For nonholonomic systems, the trajectory-tracking problem is easier to solve and more natural than posture stabilization. According to Brockett`s condition asymptotic stability of a nonholonomic system to a fixed posture is only possible with a time-varying or discontinues feedback. Stabilization, therefore, cannot be achieved by a continuous time-invariant feedback low. In the case of a trajectory-tracking controller a linear time-varying system is obtained by approximate linearization around the trajectory. The linearization obtained is shown to be controllable as long as the trajectory does not come to a stop, which implies that the system can be asymptotically stabilized by smooth linear or nonlinear feedback (Divya et al., 2022; Ghasemi et al., 2023).

Feedforward Control Action

Open-loop control of feedforward is an intuitive approach to steering nonholonomic systems. For a given reference trajectory $(x_r(t), y_r(t))$ defined in a time interval $t \in [0, T]$, the feedforward control law is derived. The v_r, θ_r, w_r are as following Equation 3 and 4.

$$v_r(t) = \pm\sqrt{\dot{x}_r^2(t) + \dot{y}_r^2(t)} \quad (3)$$

$$\theta_r(t) = A\,tan\,2\big(\dot{y}_r(t), \dot{x}_r(t)\big) + k\pi \tag{4}$$

By calculating the time derivative of Equation 4, the robot`s angular velocity $w_r(t)$ is obtained:

$$w_r(t) = \frac{\dot{x}_r(t)\ddot{y}_r(t) - \dot{y}_r(t)\ddot{x}_r(t)}{\dot{x}_r^2(t) + \dot{y}_r^2(t)} = v_r(t)k(t) \tag{5}$$

It should be mentioned that the $v_r(t)$ should not be zero. If for some time the tangential velocity $w_r(t)$.The angle $\theta_r(t)$ must be given explicitly.

Error-model

The calculated robot inputs drive the robot on a desired path only if there are no disturbances and no initial stat errors. However, when the robot is controlled to drive on a reference path, it usually has some state-following error. The state-tracking error $e(t) = [e_1(t)\ e_2(t)\ e_3(t)]^T$expressed in the frame of the real robot, as shown in Figure.3, reads Equation 2.

$$e = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} (\mathbf{q}_r - \mathbf{q}) \tag{6}$$

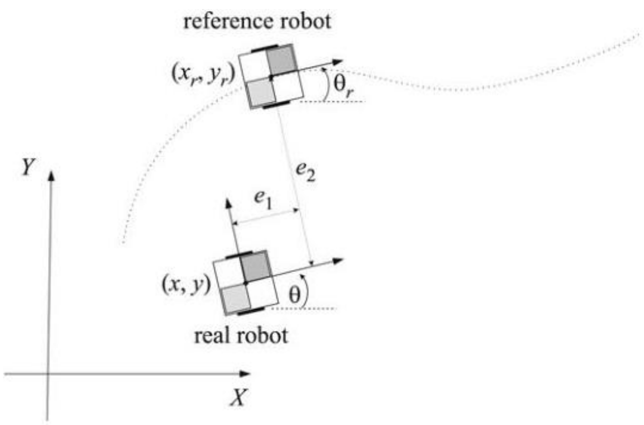

Figure 3 Error-Analysis

Taking into account the robot`s kinematics (2) and deriving the relations (6) the following kinematic model is obtained:

$$\dot{e} = \begin{bmatrix} \cos e_3 & 0 \\ \sin e_3 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_r \\ w_r \end{bmatrix} + \begin{bmatrix} -1 & e_2 \\ 0 & -e_1 \\ 0 & -1 \end{bmatrix} u \tag{7}$$

Where $u = [v \quad w]^T$ is the velocity input vector. v_r and w_r are already defined in Equation 3 and Equation 5. We can also divide the u into 2 parts, the u_F and the u_B:

$$u = u_F + u_B \tag{8}$$

Where u_F the feedforward input vector, is obtained by a nonlinear transformation of the reference inputs $u_F = [vcose_3 \quad w_r]^T$, and the feedback input vector is $u_B = [u_{B1} \quad u_{B2}]^T$.

Using relation Equation 8 and rewriting Equation 7 results in the following tracking-error models:

$$\dot{e} = \begin{bmatrix} 0 & w_r & 0 \\ -w_r & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} e + \begin{bmatrix} 0 \\ \sin\, e_3 \\ 0 \end{bmatrix} v_r + \begin{bmatrix} -1 & 0 \\ 0 & 0 \\ 0 & -1 \end{bmatrix} u_B \tag{9}$$

By linearizing the error dynamics Equation 9 around the reference trajectory ($e_1 = e_2 = e_3 = 0, u_{B1} = u_{B2} = 0$), the following linear model results:

$$\dot{e} = \begin{bmatrix} 0 & w_r & 0 \\ -w_r & 0 & v_r \\ 0 & 0 & 0 \end{bmatrix} e + \begin{bmatrix} -1 & 0 \\ 0 & 0 \\ 0 & -1 \end{bmatrix} u_B \tag{10}$$

The Equation 10, which is in the state-space form, $\dot{e} = Ae + Bu$, implies us that we can research the controllability of the system by calculating the rank:

$$[B \quad AB \quad A^2B] \tag{11}$$

Apparently, if either w_r or v_r is nonzero, the rank of Equation 11 is 3 and the system is controllable. In this case, it is possible to stabilize the system with smooth static feedback.

Use MATLAB to find the characteristic value of the matrix A:

$$\lambda = \begin{bmatrix} 0 \\ wi \\ -wi \end{bmatrix} \tag{12}$$

Considering w is nonzero, the system can have asymptotic stability by placing the right pole of the state-space or putting the fit feedback of the outputs.

When it comes to a condition that in the time cycle from 0 to T, there exists a t_i that both $\mathcal{V}_r$ and $\mathcal{W}_r$ is zero. Which means robots need to stop at a moment, the asymptotic stability therefore cannot be concluded.

Feedback Modification

The u_B in Equation 10 is the feedback control signal which return the state of e to the inputs by a linear transform k Equation 3.

$$u_B = ke \tag{13}$$

and, Equation 4.

$$k = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \end{bmatrix} \tag{14}$$

Put the Equation 13 and Equation 14, we will get the zero-inputs state-space Equation 5:

$$\dot{e} = \begin{bmatrix} -k_{11} & w_r - k_{12} & -k_{13} \\ -w_r & 0 & v_r \\ k_{21} & k_{22} & k_{23} \end{bmatrix} e \tag{15}$$

From the Equation 15, we can find that the state-space-transformation matrix is a time-varying matrix. To conclude whether the error system can get asymptotic stability is complex and difficult.

Use MATLAB and calculate the pole of Equation.15.

$$\lambda = \begin{pmatrix} \frac{k_{23}}{3} - \frac{k_{11}}{3} + \sigma_2 + \frac{\sigma_3}{\sigma_2} & 0 & 0 \\ 0 & \frac{k_{23}}{3} - \frac{k_{11}}{3} - \frac{\sigma_2}{2} - \frac{\sigma_3}{2\sigma_2} - \sigma_1 & 0 \\ 0 & 0 & \frac{k_{23}}{3} - \frac{k_{11}}{3} - \frac{\sigma_2}{2} - \frac{\sigma_3}{2\sigma_2} + \sigma_1 \end{pmatrix} \tag{16}$$

Where σ_2 and σ_3 are polynomial of the elements in $\boldsymbol{k}$, v and w. And σ_1 represents the imaginary part.

To ensure each moment, the poles of the error-model is in the negative part plane, $\boldsymbol{k}$ must be time-varying matrix. The aim is to find a const set of $\boldsymbol{k}$ to get better robustness. It should be pointed out that for the continuous system, even the pole of the ss system is positive , it doesn`t mean the system is unstable, thus a possibility for get a simple feedback matrix $\boldsymbol{k}$ is considerable.

For better discussion, we name the pole of the system as:

$$\lambda_1 = \frac{k_{23}}{3} - \frac{k_{11}}{3} + \sigma_2 + \frac{\sigma_3}{\sigma_2}$$
$$\lambda_2 = \frac{k_{23}}{3} - \frac{k_{11}}{3} - \frac{\sigma_2}{2} - \frac{\sigma_3}{2\sigma_2} - \sigma_1 \qquad (17)$$
$$\lambda_2 = \frac{k_{23}}{3} - \frac{k_{11}}{3} - \frac{\sigma_2}{2} - \frac{\sigma_3}{2\sigma_2} + \sigma_1$$

Either k_{11} and k_{23} should better not be zero at same time. If we assume that k_{23}, then the real part of the pole is:

$$-\frac{k_{11}}{3} + t$$
$$-\frac{k_{11}}{3} - \frac{t}{2}$$
$$-\frac{k_{11}}{3} - \frac{t}{2}$$

where, t is the time-varying part, and t equals

$$\sigma_2 + \frac{\sigma_3}{\sigma_2}$$

Therefore, we get the range of k_{11}:

$$k_{11} > 3t, \qquad (\text{ if t} > 0)$$
$$k_{11} > -\frac{3}{2}t, \qquad (\text{ if t} < 0) \qquad (15)$$

Conclusions

In this paper, a state-space equation of the error is introduced. The error is made by actual posture and reference postures. By splitting the inputs u into u_F and u_B, we transform the whole equation to a zero-input state-space system. With the optimization of the feedback, we can minimum the errors. On summary, the error-model tells us that by optimizing time-varying feedback matrix, we can make sure that the error will be asymptotic stable and close to zero. Therefore, by adding this control law to the controller, an approach to make mobile robot follow the planned trajectory will be obtained.

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