Portfolio Optimization: Application and Comparison of Markowitz Model and Single Index Model on LQ45 Stocks in Indonesia Stock Exchange

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Abstract: This paper examines the optimization of an Indonesian stock portfolio using two models: the Markowitz Model (Mean-Variance Model) and the Single Index Model. The data comprises historical returns of LQ 45 stocks from January 2016 to December 2021. The focus is on selecting stocks for the portfolio and determining their weights based on the two models. The study compares the performance of both optimized portfolios with the LQ45 Index benchmark, IHSG market, and each other using Sharpe and Treynor measurements. The paper tests whether the stock composition of the optimized portfolio from both models successfully and consistently generates a better performance in the future (1 January 2022 – 31 December 2022) and (1 January 2023 – 31 December 2023) compared to both LQ45 and IHSG. The results reveal a notable contrast in portfolio performance between 2022 and 2023. In 2022, both the Markowitz and Single Index portfolios exhibited remarkable returns, surpassing LQ 45 and IHSG. However, in 2023, both portfolios experienced substantial underperformance, with negative returns and unfavorable risk-adjusted metrics. These findings underscore the dynamic nature of financial markets and the need for continuous portfolio monitoring and adaptation. Investors are encouraged to reevaluate their portfolio strategies in response to changing market conditions. The study contributes valuable insights into the temporal variability of optimized portfolios and their sensitivity to evolving market dynamics.

Keywords: Indonesian stocks, portfolio optimization, Markowitz Model, Single Index Model, LQ 45, IHSG, returns, risk metrics.

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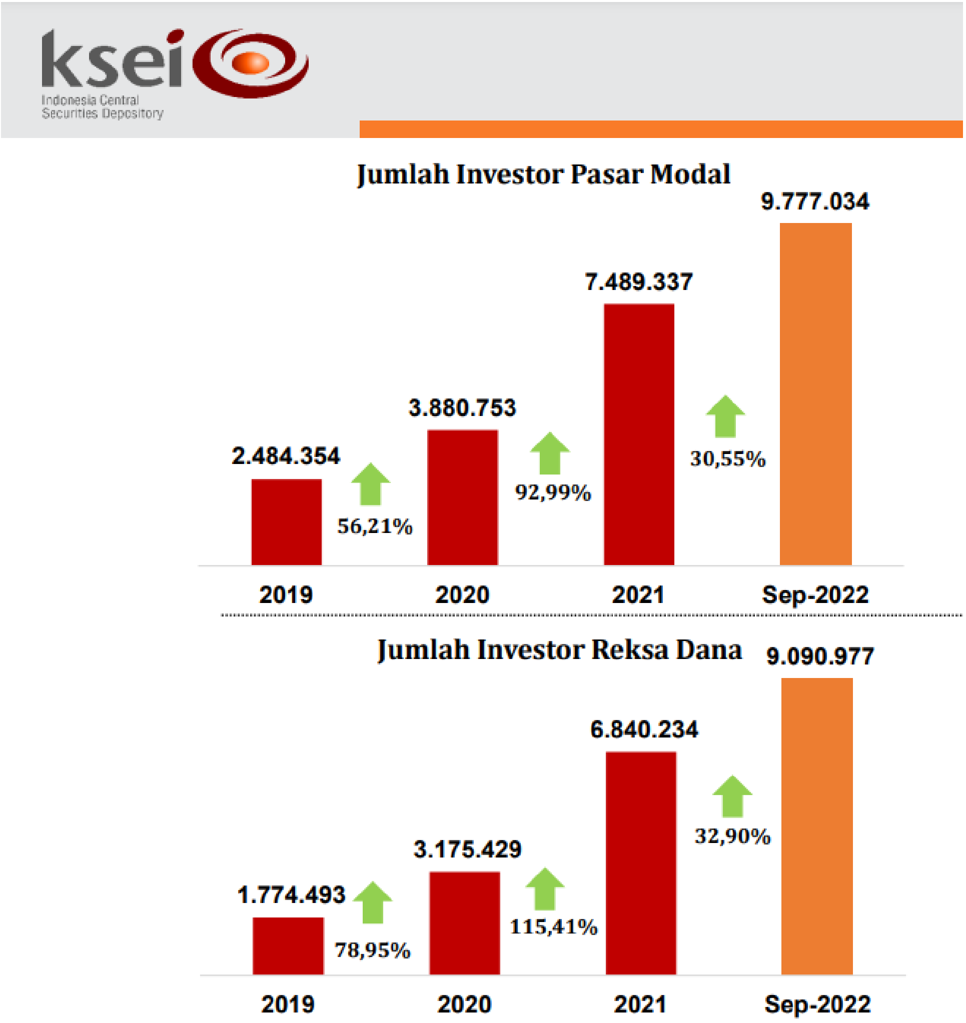
Received: December 8, 2023; Accepted: February 11, 2024; Publication: February 13, 2024

Introduction

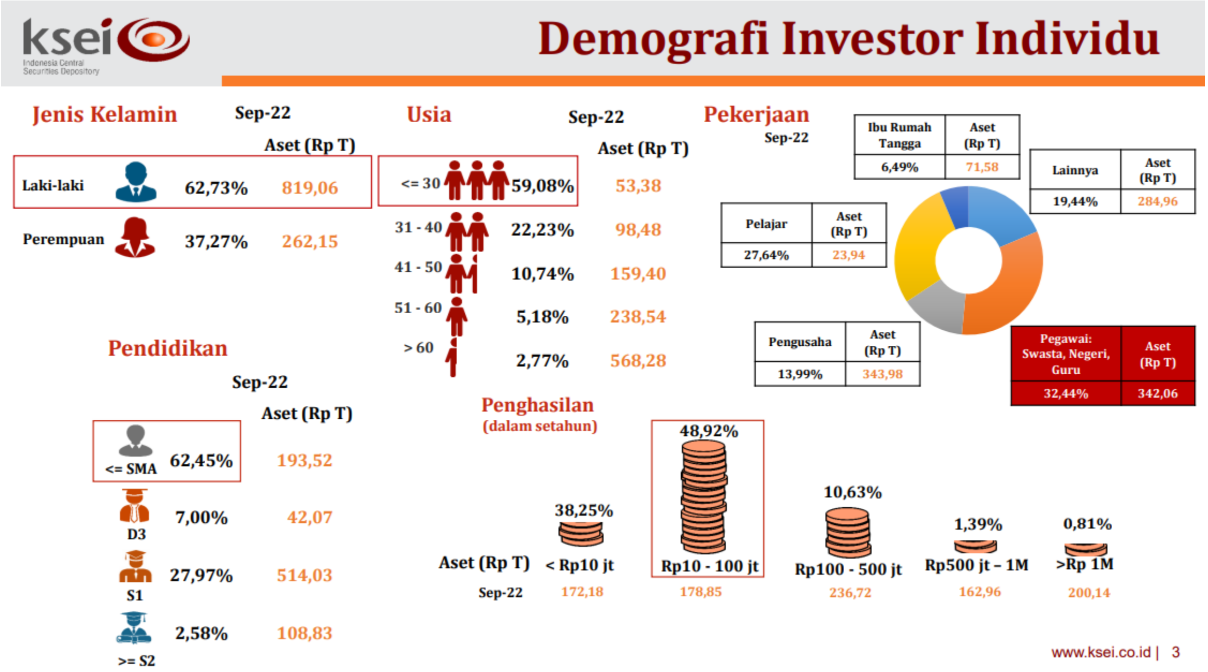
The Indonesian capital market has experienced a remarkable surge in investors, evident in the report from the Indonesian Central Securities Depository (KSEI). As of September 2022, the number of retail investors in the capital market surpassed 9.77 million, marking a notable 30.55% increase compared to the previous year-end figures.



Source: https://www.ksei.co.id/

Figure 1 Capital Market and Mutual Fund Investors Growth in Indonesia

Notably, millennials, individuals under the age of 30, constitute a majority, accounting for 59.08% of the total investor base. Concurrently, mutual fund investors reached 9.09 million by the same period. Mutual funds serve as an accessible investment avenue, particularly for novice investors, managed by professional fund managers who allocate pooled funds into diverse securities portfolios (Sun, 2010). However, the advent of information technology has democratized investment knowledge, empowering individuals to engage directly in stock selection and portfolio construction instead of solely relying on fund managers.



Source: https://www.ksei.co.id/

Figure 2 F Demographics of Individual Investors in Indonesia

Harry Markowitz's pioneering work in 1952 introduced a one-period portfolio selection model aimed at maximizing returns for a given level of risk. The Markowitz model underscores the importance of diversification to mitigate risk, encapsulated in the adage "Don't put all your eggs in the same basket" (Tandelilin, 2010). Yet, this model has limitations, such as the need for extensive covariance/correlation matrix estimates and lacking guidelines for forecasting security risk premiums. In response, William Sharpe proposed index models, notably the Single Index model, which simplifies covariance matrix estimation and delineates risk into systematic and firm-specific components (Bodie et al., 2014). These models offer insights into diversification's efficacy and enhance portfolio analysis, thus warranting a comparison with the Markowitz model in the context of portfolio optimization.

The portfolio selection process, as delineated by Markowitz (1952), necessitates research, experience, and confidence in future instrument performance. However, calculating portfolio risk in the Markowitz model grows increasingly intricate with additional assets, requiring covariance and variance estimates. Conversely, the Single Index model streamlines risk into market and company-specific components, offering a simplified approach to portfolio risk assessment. The research problem seeks to aid investors in selecting the most suitable portfolio model that outperforms the market. This study confines data analysis to specific parameters: Monthly return data of instruments from January 2016 to December 2021. Analysis focuses on LQ-45 stocks. Stock investment pertains to short, medium, and long-term trading strategies aimed at capital gains and dividends, with LQ-45 stocks as the chosen investment unit.

Building upon the research background, this study addresses the following queries: Can the Markowitz model yield optimal portfolio returns and risks? Can the Single Index model achieve optimal portfolio returns and risks? How does the performance of the Markowitz model compare to that of the Single Index model?

Aligned with investor goals of risk mitigation and return maximization, this study aims to: Analyze the formation of optimal portfolios using the Markowitz model. Assess optimal portfolio formation using the Single Index model. Compare return and risk across portfolios constructed via the Markowitz and Single Index models on the Indonesia Stock Exchange's LQ 45 Index.

This research offers utility to various stakeholders. Indonesian investors can leverage findings for informed portfolio management decisions. Contributions to academic knowledge in investment and portfolio management. Portfolio managers can integrate optimization methods to enhance portfolio performance.

The structure of this paper comprises: Introduction: Provides background, problem statement, objectives, benefits, boundaries, and structure of the analysis. Theoretical Foundation: Reviews literature on management, investment, stocks, market indices, and portfolio strategies, with a focus on the Markowitz and Single Index models. Research Methods: Describes data collection, processing techniques, portfolio evaluation, and research approach. Analysis and Discussion: Presents data processing results and analysis, addressing risk assessment and optimal portfolio formation. Conclusion and Suggestions: Summarizes research findings and offers recommendations for investors.

Literature Review

Theoretical Basis

Investment serves as a fundamental pillar in the financial landscape, encapsulating various commitments of funds with the anticipation of yielding positive returns. Fischer & Jordan (1995) describe investment as a commitment made in the expectation of a positive rate of return, echoing Francis' (1991) sentiment that it involves committing money with the anticipation of generating additional wealth. Tandelilin (2010) further elaborates that investment entails allocating funds or resources in the present with the objective of accruing benefits in the future, whether through capital appreciation or dividends. This deferred consumption to enable future gains underscores the essence of investment, as elucidated by Halim (2003) and Ahmad (2004).

Stocks, also known as shares, stand as one of the most prominent instruments in the capital market due to their potential for lucrative returns. Defined as securities traded in the capital market, stocks represent ownership in a company and confer rights to the company's assets and prospects (Husnan, 2015). Darmadji & Fakhruddin (2015) delineate various types of stocks, including ordinary shares and preferred stock, each with distinct characteristics regarding dividend entitlements and asset claims.

Integral to understanding stock market dynamics is the concept of stock market indices, which serve as principal indicators of price movements. These indices, such as the LQ45 index in Indonesia, play multifaceted roles, serving as benchmarks for portfolio performance and facilitating the formation of investment strategies (Darmadji & Fakhruddin, 2015). The LQ45 index comprises 45 stocks with high liquidity and market capitalization, selected based on stringent criteria including transaction volume and financial stability (Tandelilin, 2010).

In pursuit of investment objectives, investors are primarily motivated by the prospect of maximizing returns while navigating associated risks. The notion of return encompasses both yield and capital gain/loss, with yield representing periodic income from investments, while capital gain/loss reflects changes in asset prices (Tandelilin, 2010). Expected return serves as a pivotal metric in investment decision-making, representing the anticipated return from investment endeavors (Jogiyanto, 2017). Moreover, portfolio return, whether realized or expected, encapsulates the aggregate returns from individual securities held within a portfolio (Halim, 2003; Jogiyanto, 2017).

Mitigating investment risk constitutes a critical aspect of portfolio management, necessitating a nuanced understanding of various risk factors. Risk is defined as the possibility of experiencing a loss, which is measured in terms of the probability (Gumanti, 2011). Halim (2003) categorizes investment risks into business risk, liquidity risk, interest rate risk, market risk, purchasing power risk, and currency risk, each posing distinct challenges to investors. Furthermore, modern portfolio theory delineates systematic risk, which pertains to market-wide fluctuations, and unsystematic risk, which is specific to individual securities and can be mitigated through diversification (Halim, 2003).

Diversification emerges as a paramount strategy for managing portfolio risk, enabling investors to achieve lower portfolio risk without compromising returns (Jones, 2014). Markowitz's diversification principles underscore the importance of optimizing portfolio composition based on covariance and correlation coefficients to minimize risk exposure (Fabozzi, 1999; Tandelilin, 2010). The Markowitz model, a cornerstone of modern portfolio theory, emphasizes the construction of diversified portfolios to maximize returns while mitigating risk (Markowitz, 1952). Conversely, the Single Index Model, introduced by Sharpe, offers a simplified approach by relating asset returns to market index movements, facilitating portfolio optimization based on the Excess Return to Beta (ERB) metric (Halim, 2003).

Despite their differences, both the Markowitz model and the Single Index Model aim to guide investors in constructing optimal portfolios aligned with their risk preferences and return objectives. The former emphasizes diversification to minimize risk, while the latter offers a straightforward method leveraging market index movements to inform portfolio decisions (Jogiyanto, 2017). Understanding the nuances and implications of these models is crucial for investors seeking to navigate the complex terrain of portfolio management and optimize investment outcomes.

Table 1 Markowitz & Single Index Model Differences Matrix

#	Comparison	
	Markowitz Model	**Single Index Model**
1	Doesn't take into account the risk-free assets	The risk-free assets are taken into account
2	The risk calculation is based on variance-covariance matrix	The risk calculation is based on market risk and other company specific risk
3	Investor's preference is based on expected return and risk	Considering the return of each asset on the market index return
4	Calculations tend to be complex	Simplification of the Markowitz model
5	Does not measure the systematic risk of a security or portfolio relative to market risk	Takes into account security and portfolio risk relative to market risk

Source: Jogiyanto (2017)

Previous Researches

The research landscape surrounding optimal portfolio formation spans various models and methodologies, each offering unique insights into investment decision-making. Studies such as Cai & Long (2022) and Ni (2022) delve into comparing the effectiveness of the Markowitz model and the Index model, highlighting nuances in portfolio construction. Cai & Long (2022) emphasize the sharper efficient frontier of the Markowitz model, indicating its suitability for maximizing returns, while Ni suggests that the Index model requires less computational effort, albeit with slightly better portfolio outcomes. Wang (2022) extends this exploration by investigating the impact of external factors like COVID-19 on portfolio performance, revealing differential outcomes under various constraints for both models.

Risk-averse investors find solace in methodologies like mean-variance analysis, as demonstrated by Liu (2022), who showcases the applicability of this approach in quantifying expected returns and acceptable risk levels. Abdullah & Ishak (2021) corroborate these findings by illustrating the superiority of optimal portfolios over naive strategies during different financial periods, using the Markowitz mean-variance approach. Additionally, the integration of fundamental analysis with mean-variance optimization, as seen in Lyle & Yohn (2020) and Bielstein & Hanauer (2017), offers a robust framework for achieving high Sharpe ratios and outperforming traditional approaches.

Performance evaluation of portfolio models further illuminates the landscape, with studies like Sarker (2015) favoring the Constant Correlation Model over others, while Aisyah & Nasution (2021) provide insights into effective portfolio construction during crises using the Single Index Model. Soraya (2021) introduces alternative approaches by combining Graham's stock selection model with the Markowitz model, showcasing superior performance compared to index portfolios.

Amidst the plethora of methodologies, the Single Index Model stands out in certain contexts, as evidenced by studies like Utami et al. (2021), which determine optimal portfolios during crises periods, and Putra & Dana (2020), which finds better performance using the Single Index Model compared to the Markowitz model. However, the performance disparities between models are not always significant, as observed by Qu et al. (2021), indicating the need for careful consideration of constraints and objectives in portfolio construction.

Furthermore, studies such as Abdullah & Ishak (2021) and Amaroh & Nasichah (2021) provide empirical evidence on the efficacy of Markowitz mean-variance analysis in diversification strategies and risk-return analysis, particularly during different financial periods and within specific market indices. Similarly, Dewi (2021), Hendra et al. (2021), Agustina & Sari (2019) explore optimal portfolio formation using the Markowitz model, demonstrating the practical application of this approach in maximizing portfolio returns while managing risk.

The exploration of alternative portfolio optimization techniques is also evident in research such as Soraya (2021), which combines Graham's stock selection model with the Markowitz model, and Setyantho & Wibowo (2019), which utilizes financial indexes to develop optimal portfolios. These studies highlight the versatility of portfolio optimization methodologies and the importance of tailoring approaches to specific market conditions and investor preferences.

In summary, the research landscape on optimal portfolio formation is diverse, offering a rich tapestry of methodologies and insights. While the Markowitz model remains a cornerstone in portfolio theory, alternative approaches like the Single Index Model provide viable alternatives, especially in specific contexts such as crisis periods. The efficacy of each model ultimately depends on factors such as computational efficiency, risk tolerance, and investment objectives, highlighting the importance of tailored approaches in portfolio management.

This comprehensive overview of portfolio optimization research is further enriched by additional studies. Thomas et al. (2017) examined optimal portfolios in trusted Indonesian companies using the Single Index Model, finding insights into portfolio formation strategies within specific market segments. Yuwono & Ramdhani (2017) compare portfolio formation methodologies on the Jakarta Islamic Index, suggesting no significant difference in returns between the Markowitz model and the Single Index Model. Azizah et al. (2017a) explore optimal portfolio formation on the Jakarta Islamic Index, revealing similar results between the Single Index Model and the Markowitz Model. Finally, Azizah et al. (2017b) and Setyantho & Wibowo (2019) contribute to the discourse by investigating optimal portfolio formation on the Jakarta Islamic Index, highlighting varying expected returns and risk levels under different models and methodologies.

Hypotheses Development

Traditionally, many research studies in portfolio optimization have focused on analyzing historical data to form optimal portfolios without testing their performance beyond the data gathering period. In contrast, this research aims to bridge this gap by evaluating the performance of optimized portfolios after the data gathering period. Based on the problem formulation and the research objectives, the hypotheses of this study are developed as follows:

- H01: The level of risk and return obtained in the Markowitz optimal portfolio formation model, using historical 2016-2021 data under free constraint, is hypothesized to be consistently better compared to that of the index (LQ45) and the market (IHSG) during the subsequent 2022 and 2023 period. Conversely, the alternative hypothesis Ha1 suggests that the Markowitz model may yield worse risk and return outcomes compared to the index and the market during the subsequent period.
- H02: Similarly, the level of risk and return obtained in the Single Index optimal portfolio formation model, based on historical 2016-2021 data, is expected to be consistently better compared to that of the index (LQ45) and the market (IHSG) during the subsequent 2022 and 2023 period (H02). Conversely, the alternative hypothesis Ha2 posits that the Single Index model may lead to inferior risk and return outcomes compared to the index and the market during the subsequent period.
- H03: Lastly, the level of risk and return obtained in the Markowitz optimal portfolio formation model, using historical 2016-2021 data under free constraint, is anticipated to be better compared to the level of risk and return obtained in the Single Index optimal portfolio formation model during the subsequent 2022 and 2023 period (H03). Conversely, the alternative hypothesis Ha3 suggests that the Markowitz model

may result in worse risk and return outcomes compared to the Single Index model during the subsequent period.

Research Method

This study adopts a descriptive research approach, aiming to elucidate the characteristics and nature of a specific phenomenon (Umar, 2003). The data utilized in this study are secondary in nature, sourced from www.idx.com, comprising information on companies listed in the LQ 45 Index and historical monthly closing prices. Secondary data, as described by Sugiyono (2013), are obtained indirectly from original sources through documents or other intermediaries, aligning with the data collection approach of this study.

Population and Sampling Technique

The population under study encompasses all stocks included in the LQ 45 Index on the Indonesia Stock Exchange from January 2016 to December 2021. As defined by Sugiyono (2013), the population refers to the generalization of objects or subjects with specific qualities and characteristics selected for study. The population consists of 68 companies based on observations of the LQ 45 Index during the specified timeframe.

To select the research sample, a purposive sampling method was employed based on specific criteria. The criteria include the consecutive listing of companies in the LQ 45 Index over eleven observation periods ranging from August 2015 to January 2022. Companies meeting these criteria were considered for inclusion in the sample. Upon application of the sampling criteria, 42 companies were identified as not consistently listed in the LQ 45 Index throughout the observation periods. Consequently, 26 samples were selected for portfolio formation analysis, representing entities that met the specified criteria consistently over the study period. These samples in Table 3 were chosen to ensure relevance and consistency in the analysis of portfolio formation strategies.

Table 3 Research Sample

No	Code	Company Name	No	Code	Company Name
1	ADRO	Adaro Energy Tbk	14	INTP	Indocement Tunggal Prakarsa Tbk
2	AKRA	AKR Corporindo Tbk	15	JSMR	Jasa Marga (Persero) Tbk
3	ASII	Astra International Tbk	16	KLBF	Kalbe Farma Tbk
4	BBCA	Bank Central Asia Tbk	17	MNCN	Media Nusantara Citra Tbk
5	BBNI	Bank Negara Indonesia (Persero) Tbk	18	PGAS	Perusahaan Gas Negara (Persero) Tbk
6	BBRI	Bank Rakyat Indonesia (Persero) Tbk	19	PTBA	Bukit Asam Tbk
7	BBTN	Bank Tabungan Negara (Persero) Tbk	20	PTPP	PP (Persero) Tbk
8	BMRI	Bank Mandiri (Persero) Tbk	21	PWON	Pakuwon Jati Tbk
9	BSDE	Bumi Serpong Damai Tbk	22	SMGR	Semen Indonesia (Persero) Tbk
10	GGRM	Gudang Garam Tbk	23	TLKM	Telekomunikasi Indonesia (Persero) Tbk
11	ICBP	Indofood CBP Sukses Makmur Tbk	24	UNTR	United Tractors Tbk
12	INCO	Vale Indonesia Tbk	25	UNVR	Unilever Indonesia Tbk
13	INDF	Indofood Sukses Makmur Tbk	26	WIKA	Wijaya Karya (Persero) Tbk

Variables and Measurements

The measurement and operationalization of variables in this study, based on Jogiyanto (2017), employ the Markowitz Model and the Single Index Model.

Under the Markowitz Model:

1. Rate of return (Rit) for each stock is calculated by subtracting the previous month's shares from the current period's shares and then dividing by the current stock price.
2. Expected return (E(Ri)) is determined by first calculating the individual stock return value (Ri) for candidate stock portfolios based on weekly stock prices over the observation period.
3. Risk of each stock (σ) is calculated to measure investment risk and assess potential deviations from expected values.
4. A combined portfolio is computed.
5. Investment weight of the fund is determined based on investor preferences, aiming to balance profit and risk.
6. Expected return level is calculated from the formed portfolio.
7. Correlation coefficient of stock prices between companies is computed to understand portfolio risk diversification.
8. Stock risk from a portfolio is assessed to identify potential stocks for the optimal portfolio, characterized by an efficient balance between expected return and risk.

Under the Single Index Model:

1. Realized return (Rm) is determined as the percentage change in market share prices from the current month to the previous month.
2. Expected market return (E(Rm)) is calculated by averaging the percentage of return on market realization.
3. Market risk/Variance ($\sigma 2$) is the square of the standard deviation used to measure market risk.
4. Beta (β) serves as a systematic risk measure relative to market risk.
5. Alpha (α) compares individual stock return expectations with market expectations.
6. Variance ($\sigma 2$) captures the residual error variance of individual stocks.
7. Excess Return to Beta (ERB) calculates portfolio performance relative to undiversified risk.
8. Values such as (Ai) and (Bi) are calculated for each stock to determine proportionate funds and portfolio characteristics.
9. Limiting Point (Ci) is the value (C) for the i-th stock which is calculated from the accumulated values (A1) to (Ai) and values (B1) to (Bi).

10. Proportion of funds (Xi) and percentage of fund processing (Wi) determine investment allocation.
11. Beta portfolio ($\beta\rho$) and Alpha portfolio ($\alpha\rho$) represent individual portfolio characteristics.
12. Expected return (E(Rp)) is derived from weighted values of individual stock expected returns.
13. Portfolio variance ($\sigma 2$) reflects the weighted average of individual stock variances, providing a comprehensive framework for evaluating and optimizing portfolio performance.

The operational variables of the Markowitz Model and Single Index Model can be seen in Table 4 and Table 5.

Table 4 Variable Operationalization – Markowitz Model

#	Analysis Concept	Indicator	Scale
1	Calculating the rate of return (profit) for each stock (Rit)	a. Current month's closing price b. Closing price of the previous month's shares c. Share dividends received on shares i	Ratio
2	Calculating the expected return (profit expected) of each share (E(Ri))	a. Realized total return b. The number of observations	Ratio
3	Calculating the risk of each stock (σ)	a. Realized total return b. Expected return c. The number of observations d. Stock variance (Var(Ri))	Ratio
4	Calculating a portfolio combination	a. The stocks included in the sample are combined	Ratio
5	Determine the investment weight of the fund	a. Initial Fund b. Return	Ratio
6	Calculating the expected return (expected profit level)	a. Expected return b. Weight of funds to be invested in shares A c. Weight of funds to be invested in shares B d. Expected return on A stock e. Expected return on Stock B	Ratio
7	Calculating the correlation coefficient of stock prices between companies	a. Number of Observations b. Expected return on stock A c. Expected return on stock B	Ratio
8	Calculating the risk of a stock portfolio	a. Portfolio variant b. Portfolio stock risk c. The risk of shares A, B d. The weight of funds invested in shares A e. The weight of funds invested in shares B	Ratio

Source: Jogiyanto (2017)

Table 5 Variable Operationalization – Single Index Model

#	Analysis Concept	Indicator	Scale
1	Market realization return (Rm)	a. Current month closing price LQ 45 Index b. Previous month closing price LQ 45 Index	Ratio
2	Calculating returns market expectations E(Rm)	a. Market realizaed total return b. Number of observations	Ratio
3	Market risk/variance ($\sigma 2$)	a. Market realized total return b. Expectated market return c. Number of observations	Ratio
4	Beta β, measures the systematic risk of a security or portfolio relative to market risk	a. Realized return of individual stock b. Expected individual stock return c. Realized market return d. Expected market return	Ratio
5	Alpha (α) Intercept comparison of return expectations of individual stocks with returns market expectations	a. Expected Return Individual stock b. Individual stock beta c. Expected market return	Ratio
6	Variance ($\sigma 2$) of the Residual ei Individual stock errors which is also unique or unsystematic risk	a. Individual stock variance b. Individual stock betas c. Market variances	Ratio
7	Excess Return to Beta (ERB) Calculation to determine the optimal portfolio which measures the excess return relative to a unit of risk that is not diversified measured by beta	a. Expected return Individual stock b. Risk Free Asset Return c. Individual Stock Beta	Ratio
8	Ai and Bi The value of Ai is calculated to obtain the value of Bj. The value of both is used for get the Ci value	a. Expected return on Individual Stock b. Risk-free Assets Return c. Individual stock beta d. Individual stock Variance Error	Ratio
9	Limiting Point (Ci), the value of C for the i-th stock calculated from the accumulated values of A1 to with Ai and values B1 through Bi	a. Market variances b. Ai value c. Bi value	Ratio
10	Proportion of Funds (Xi), Percentage of proportion of funds (Wi)	a. Individual stock beta b. Individual stock Variance Error c. Excess Retrun to Beta d. Cut Off Points	Ratio
11	Beta Portfolio Bp, Individual Beta of each incoming share in the optimal portfolio	a. Individual stock beta b. Percentage Proportion of Funds	Ratio
12	Alpha Portfolio (αP), individual Alpha of each stock included optimal portfolio	a. Individual stock Alpha b. Percentage Proportion of Funds	Ratio

#	Analysis Concept	Indicator	Scale
13	Portfolio Expected Return (E(Rp), weighted average of the individual expected return of each stock in the portfolio optimal	a. Alpha Portfolio b. Beta Portfolio c. Market Exposure Return	Ratio
14	Portfolio Variance (σp), the weighted average of the individual variance of each forming stock portfolio	a. Beta Portfolio b. Market Variances c. Percentage Proportion of Funds d. Individual stock Variance Error	Ratio

Source: Jogiyanto (2017)

Data Analysis Method

In this study, data collection techniques primarily involve the documentation method, which entails gathering relevant articles, journals, and books to acquire theories pertinent to the research focus. Additionally, data is collected through the official website of the Indonesia Stock Exchange (www.idx.co.id), including information on companies listed on the LQ 45 Index, monthly closing prices of sample companies, the closing price of the LQ 45 Index, and risk-free assets from Bank Indonesia.

For data analysis, the Markowitz Model and the Single Index Model are employed to determine the types of stocks to be included in each optimal portfolio model. The calculations are conducted using the Microsoft Office Excel program. The analysis spans 60 months, from January 2016 to December 2021, utilizing a sample of 26 stocks obtained through the sampling technique. The stages in data analysis are outlined in Table 6 for the Markowitz Model, and Table 7 for the Single Index Model.

Table 6 Data Analysis – Markowitz Model

#	Analysis Concept	Calculation	Remark
1	Calculate rate of return of every stocks.	$R_{it} = \frac{P_{it} - P_{i\,(t-1)} + D_t}{P_{i\,(t-1)}}$	R_{it} = Rate of retun of i at period t $P_{i\,(t-1)}$ = Stock price at the beginning period P_{it} = Stock price at the end of period D_t = Dividend received of i
2	Calculate expected return of every stocks.	$E_{Ri} = \frac{\sum_{t=1}^{N} R_{it}}{N}$	E_{Ri} = Expected rate of return of stock i R_{it} = Return of i at period i N = Number of observation time interval
3	Calculate risk of every stocks.	$\sigma_i = \sqrt{\frac{\sum_{i=1}^{N}[R_{it} - \overline{R_{it}}]^2}{N}}$	σ_i = Individual stock risk (standard deviation) R_{it} = Rate of retun of i at period t N = Number of observation time interval
4	Calculate portfolio combination	$C(r,n) = \frac{n!}{r!\ (n - r)!}$	$C(r,n)$ = Number of combinations $n!$ = Total number of objects in the set r! =Number of choosing objects from the set

#	Analysis Concept	Calculation	Remark
5	Determine the investment fund weight.	1) $\sum_{i=1}^{N} = 1$ 2) $1 \geq w_i \geq 0, \quad i = 1 \dots n$ 3) $\sum_{i=1}^{n} w_i . R_i = R_P$	w_i = Fund weight R_i = Return of i R_P = Portfolio return
6	Calculate expected return of portfolio	$E(R_p) = X_A . E(R_A) + X_B . E(R_B)$	$E(R_p)$ = Portfolio expected return X_A = Fund weight in stock A X_B = Fund weight in stock B $E(R_A)$ = Expected return of stock A $E(R_B)$ = Expected return of stock B
7	Calculate coefficient of correlation between stocks	$\rho AB = \frac{N \sum AB - \sum A \sum B}{\sqrt{\{[N \sum A^2 . \sum(A)^2][N \sum B^2 . \sum(B)^2]\}}}$	N = Number of observation time interval A = Rate of return stock A B = Rate of return stock B
8	Calculate portfolio risk	$\sigma_P^2 = X_A^2 \sigma_A^2 + X_B^2 \sigma_B^2 + 2 X_A X_B \rho AB$ and $\sigma_P = \sqrt{\sigma_P^2}$	σ_P^2 = Portfolio variance σ_P = Portfolio risk (standard deviation) σ A, σ B= Risk of A, B XA = Fund weight in A XB = Fund weight in B

Source: Jogiyanto (2017)

Table 7 Data Analysis – Single Index Model

#	Analysis Concept	Calculation	Remark
1	Market Realized Return (Rm)	$R_m = \frac{LQ45_t - LQ45_{(t-1)} + D_t}{LQ45_{(t-1)}}$	R_m= Market Return $LQ45_t$ = Current Month LQ 45 Index Closing Price $LQ45_{(t-1)}$= Previous Month LQ 45 Index Closing Price
2	Calculating Market Expected Return E(Rm)	$E_{(Rm)} = \frac{\sum R_m}{n}$	$E_{(Rm)}$ = Expected market return $\sum R_m$ = Total market return n = Number of observations
3	Market Risk / Variance ($\sigma 2$) m	$\sigma_m^2 = \sum_{i=1}^{n} \frac{[(R_{mt} - E(R_m)]^2}{n}$	σ_m^2 = Market variance m R_{mt} = Return of market in month t $E(R_m)$ = Expected return of Market n = Number of observations
4	Beta β, Measures the systematic risk of a security or portfolio relative to market risk	$\beta_i = \frac{\sigma_{im}}{\sigma_i^2}$ *Or* $\beta_i = \frac{\sum_{i-1}^{n}(R_{it} - \overline{R_i})(R_{mt} - \overline{R_m})}{\sum_{i-1}^{n}(R_m - \overline{R_m})^2}$	β_i =Beta of individual stock σ_{im} =Covariance of return on stock A and market return σ_i^2 =Individual stock variance R_{it} =Return of individual stock in month t $\overline{R_i}$ =Expected Return of individual stocks R_{mt} =Return market in month t $\overline{R_m}$=Expected market return n = Number of Observations
5	Alpha (α) Intercept Comparison of expected return on individual stocks with return expected from the market	$\alpha = E(R_i) - \beta_i \, x \, E(R_m)$	α = individual stock alpha $E(R_i)$ = Expected Return of individual stock β_i = Individual stock beta $E(R_m)$ = Expected Market Return

#	Analysis Concept	Calculation	Remark
6	Variant ($\sigma 2$) of the Residual *ei* Individual stock error is also a unique or unsystematic risk	$\sigma_{ei}^2 = \sigma_i^2 - \beta_i^2 \ x \ \sigma_m^2$	σ_{ei}^2 = *Variance error of Individual shares* σ_i^2 = *Variance of Individual shares* β_i^2 = *Individual stock beta* σ_m^2 = *Market variance*
7	Excess Return to Beta (ERB) Calculation to determine the optimal portfolio which measures the excess return relative to a risk unit that not diversified as measured by beta	$ERB_i = \frac{E(R_i) - R_f}{\beta_i}$	ERB_i = Individual stock excess return to beta $E(R_i)$ = Individual stock expected return R_f =Average Return of Risk-Free Assets β_i =Induvidual Stock Beta
8	Ai and Bi The value of Ai is calculated to obtain the value of Bj. The value of the two is used to obtain the Ci value	$A_i = \frac{[E(R_i) - R_f] . \beta_i}{\sigma_{ei}^2}$ and $B_i = \frac{\beta_i^2}{\sigma_{ei}^2}$	$E(R_i)$ = Individual stock expected return R_f = Average Risk-Free Asset Return β_i = Individual Stock Beta σ_{ei}^2 = Variance error of individual stocks
9	Limiting Point (Ci), C value for the i-th stock calculated from the accumulation of values A1 to Ai and values B1 to Bi	$C_i = \frac{\sigma_m^2 \sum_{j=1}^{i} A_i}{1 + \sigma_m^2 \sum_{j=1}^{i} B_i}$	C_i= Limiting Point σ_m^2 =Market variance A_i = Alpha i B_i = Beta i
10	Proportion of Funds (Zi), Percentage of proportion of funds (Wi)	$Z_i = \frac{\beta_i}{\sigma_{ei}^2}(ERB - C^*)$ And $W_i = \frac{Z_i}{\sum Z_i}$	Z_i = Proportion of individual stock funds β_i = Individual stock betas ERB = Excess Return to Beta of individual stocks C^* = Cut Off Point W_i = Percentage of the proportion of individual stock funds $\sum Z_i$ = Total proportion of stock funds in the portfolio
11	Beta Portfolio Bp, Individual Beta of each stock included in the optimal portfolio	$\beta_p = \sum_{i=1}^{n} W_i \beta_i$	β_p = Portfolio beta W_i = Proportion of individual stock funds β_i = Individual stock beta
12	Portfolio Alpha (αP), individual Alpha of each stock included in the optimal portfolio	$\alpha_p = \sum_{i=1}^{n} W_i \alpha_i$	$\alpha \rho$ =Alpha Portfolio Wi = Proportion of Individual Stock Fund αi = individual stock alpha
13	Expected Return Portfolio (E(Rp), weighted average of the individual	$E(R_p) = \alpha_p + \beta_p(R_m)$	$E(Rp)$ = Expected Return portfolio

#	Analysis Concept	Calculation	Remark
	return expectations of each stock in the optimal portfolio		
14	Portfolio Variance (σp), the weighted average of the individual variance of each forming stock portfolio	$\sigma_p^2 = \beta_p^2 \sigma_m^2 + \sum W_i^2 \sigma_{ei}^2$	σ_p^2 = Portfolio Variance

Source: Jogiyanto (2017)

After optimized portfolios from both models are acquired, we will then use Sharpe ratio and Treynor ratio for evaluating portfolio performance, by comparing it with the market and with each other. Sharpe measures how different the risk premium is generated for each unit of risk taken. Taking into account risk, the higher the value of the Sharpe measurement, the better the portfolio performance. Measurement using the Treynor method is also based on a risk premium like Sharpe's. However, in Treynor what is used as a dividing factor is Beta which is a systematic risk or also called market risk. The higher the Treynor value, the better the portfolio performance.

Table 8 Portfolio Evaluation Measures

#	Analysis Concept	Calculation	Remark
1	Sharpe ratio	$Sharpe\ ratio = \frac{R_P - R_F}{\sigma_P}$	R_P =Average Return of Portfolio R_f =Average Return of Risk-Free Assets σ_P =Portfolio standard deviation
2	Treynor ratio	$Treynor\ ratio = \frac{R_P - R_F}{\beta_P}$	R_P =Average Return of Portfolio R_f =Average Return of Risk-Free Assets β_P =Portfolio Beta

Source: https://www.investopedia.com/

To summarize, Figure 3 outlines the sequential steps involved in this research study. Initially, historical data from January 1, 2016, to December 31, 2021, is utilized to construct optimized portfolios based on both the Markowitz and Single Index models. These optimized portfolios are constructed using data analysis techniques and are aimed at maximizing returns while managing risks effectively.

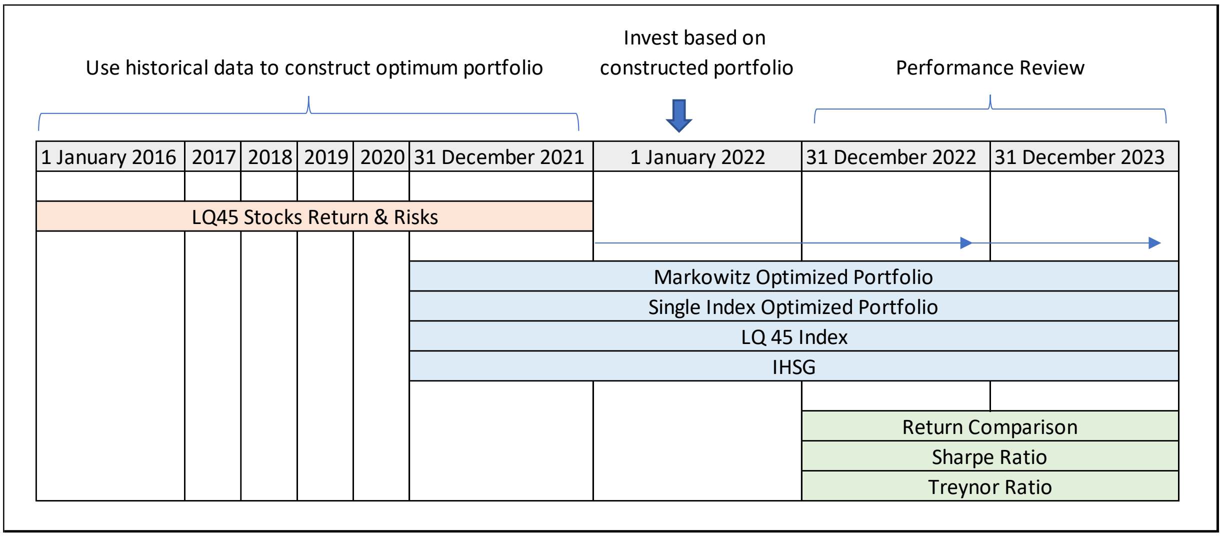


Figure 3 Research Sequence

Subsequently, the performance of these constructed portfolios is reviewed, compared, and evaluated against the LQ 45 Index and the IHSG (Indonesia Stock Exchange Composite Index) over the period from January 1, 2022, to December 31, 2023. Performance comparison is conducted based on return metrics, including Sharpe ratio and Treynor ratio, to assess the effectiveness and efficiency of the optimized portfolios in generating returns relative to the market benchmarks. This sequential process provides a structured approach to analyzing and evaluating portfolio performance in the context of historical market data.

Result and Discussion

Data Processing – Markowitz Model

The data processing phase employing the Markowitz model commenced with the utilization of historical data encompassing 26 samples from 2016 to 2021. The objective was to formulate an optimal portfolio for assessing portfolio performance.

First, the expected return (ERi) of each stock was calculated by determining the average yield over the study period. This involved dividing the total returns by the number of research months using the average function in Microsoft Excel. Monthly expected returns were derived based on fixed probability distributions. Among the 26 stocks, 7 exhibited negative average monthly returns, rendering them ineligible for inclusion in the Markowitz model-based optimal portfolio. Conversely, ADRO, INCO, and PTBA demonstrated the highest monthly average returns. Subsequently, the standard deviation (risk) of each stock was computed to assess price movement volatility. The stdevp function in Microsoft Excel facilitated this calculation, revealing that 6 stocks exhibited considerably lower standard deviation compared to others. This signified lower price movement volatility for these stocks. With the expected returns and risks determined for each stock, a return-risk profile was plotted for all samples, aiding in visualizing their performance (Figure 4).

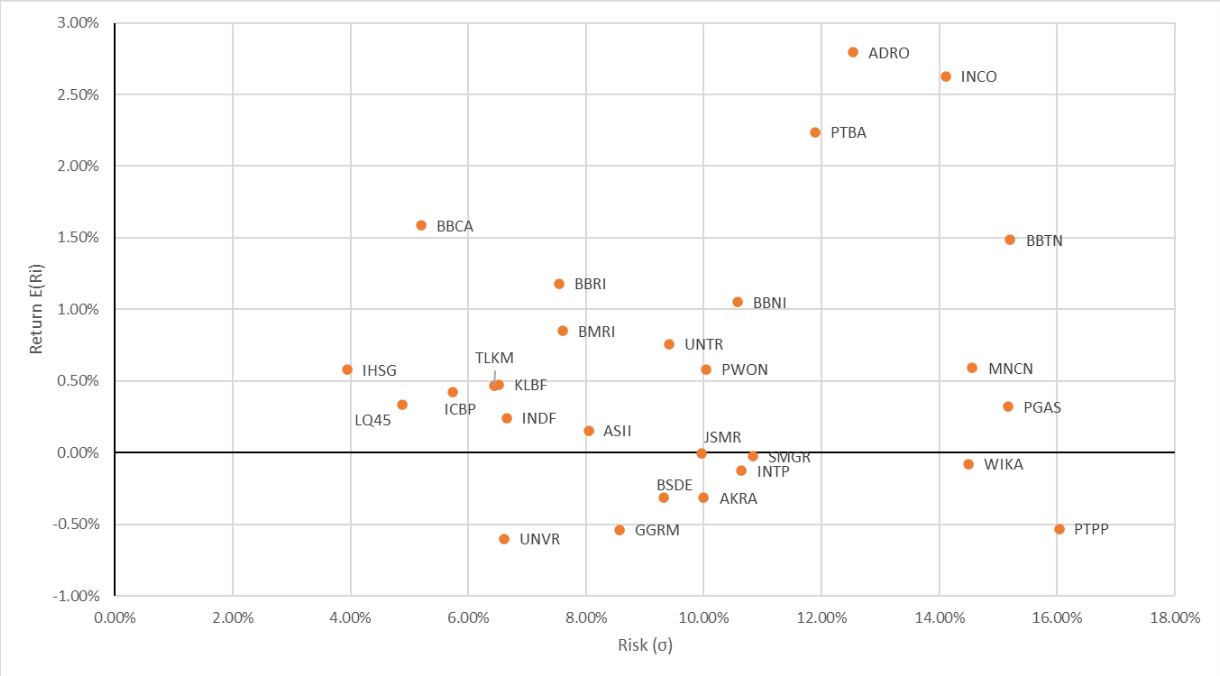


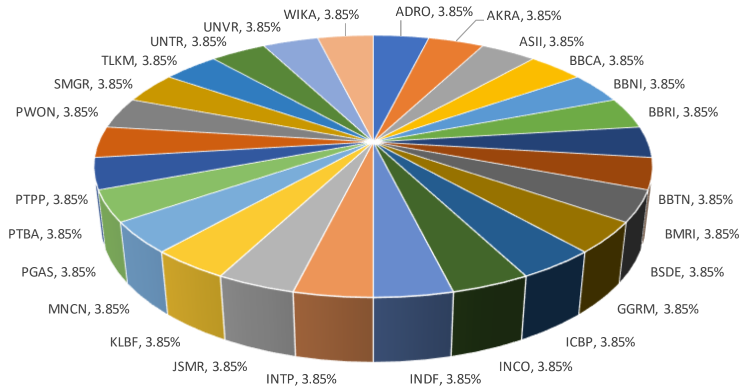
Figure 4 Risk-Return Profile Plot

Further, the variance-covariance matrix was generated to evaluate the relationship between stocks. Covariance, as a measure of how two datasets differ, was calculated to determine the degree of correlation between variables.

Once we got all the data for expected return E(Ri), risk (σ), and covariances of each stock toward another, we can form a naïve portfolio, that consisted of equally weighted fund for the 26 samples, and calculate this portfolio risk and return.

In Figure 5, an equally weighted portfolio across all 26 stocks reveals an expected return of 0.59% and a risk (standard deviation) of 6.14%. The resulting Sharpe ratio, which indicates the reward-to-volatility ratio, stands at 9.59%. However, this performance falls short compared to the IHSG, where a similar return of 0.58% is achieved with a lower risk of 3.95%, resulting in a Sharpe ratio of 14.71%. This underscores the necessity for portfolio optimization to maximize returns relative to risk.

Stock	Equally Weighted
ADRO	3.85%
AKRA	3.85%
ASII	3.85%
BBCA	3.85%
BBNI	3.85%
BBRI	3.85%
BBTN	3.85%
BMRI	3.85%
BSDE	3.85%
GGRM	3.85%
ICBP	3.85%
INCO	3.85%
INDF	3.85%
INTP	3.85%
JSMR	3.85%
KLBF	3.85%
MNCN	3.85%
PGAS	3.85%
PTBA	3.85%
PTPP	3.85%
PWON	3.85%
SMGR	3.85%
TLKM	3.85%
UNTR	3.85%
UNVR	3.85%
WIKA	3.85%
Total	**100.00%**
Return	**0.59%**
Risk	**6.14%**
Return/Risk	**9.59%**



	LQ45	**IHSG**
Return	**0.34%**	**0.58%**
Risk	**4.87%**	**3.95%**
Return/Risk	**6.92%**	**14.71%**

Figure 5 Equally Weighted Portfolio

In the pursuit of efficient portfolios, we employed Ms. Excel Solver to optimize portfolios, resulting in 22 sets of portfolios with varied risk-return profiles. Table 9 presents detailed insights into each portfolio's stock allocation, risk, return, and Sharpe ratio, with Portfolio 1 demonstrating the lowest risk and Portfolio 22 showcasing the highest return. Drawing from this dataset, the efficient frontier, optimizing portfolios with maximum returns for every given risk level, was delineated.

Table 9 Efficient Portfolios

Portfolio	Weight of Each Stock										Total Weight	Risk (σ)	Return (μ)	Return / Risk (μ/σ)
	ADRO	BBCA	GGRM	ICBP	INCO	JSMR	PTBA	TLKM	UNTR	UNVR				
Portfolio 1	**1.24%**	**24.89%**	**11.34%**	**26.68%**	**0.71%**	**1.74%**	**1.24%**	**15.22%**	**6.02%**	**10.91%**	**100.00%**	**3.58%**	**0.58%**	**16.20%**
Portfolio 2	1.94%	27.86%	10.50%	26.86%	1.31%	0.84%	1.31%	15.22%	5.18%	8.97%	100.00%	3.59%	0.67%	18.80%
Portfolio 3	2.86%	31.59%	9.32%	27.15%	2.04%	-	1.43%	15.11%	4.07%	6.45%	100.00%	3.62%	0.79%	21.95%
Portfolio 4	3.85%	35.10%	7.81%	27.61%	2.72%	-	1.62%	14.62%	2.86%	3.81%	100.00%	3.68%	0.91%	24.87%
Portfolio 5	4.88%	38.59%	6.34%	28.10%	3.41%	-	1.74%	14.20%	1.63%	1.12%	100.00%	3.76%	1.03%	27.51%
Portfolio 6	6.03%	42.73%	4.29%	27.69%	4.01%	-	2.06%	13.05%	0.14%	-	100.00%	3.87%	1.15%	29.83%
Portfolio 7	7.01%	47.42%	1.41%	26.23%	4.68%	-	2.10%	11.01%	0.13%	-	100.00%	4.01%	1.27%	31.78%
Portfolio 8	8.42%	52.23%	-	23.72%	5.58%	-	2.05%	8.00%	-	-	100.00%	4.19%	1.39%	33.32%
Portfolio 9	10.18%	57.21%	-	20.11%	6.69%	-	1.93%	3.88%	-	-	100.00%	4.41%	1.51%	34.36%
Portfolio 10	11.91%	62.15%	-	16.28%	7.82%	-	1.84%	-	-	-	100.00%	4.68%	1.63%	34.96%
Portfolio 11	**13.93%**	**66.45%**	**-**	**9.04%**	**9.20%**	**-**	**1.38%**	**-**	**-**	**-**	**100.00%**	**4.99%**	**1.75%**	**35.15%**
Portfolio 12	15.58%	70.03%	-	3.00%	10.36%	-	1.03%	-	-	-	100.00%	5.29%	1.85%	35.05%
Portfolio 13	19.68%	67.80%	-	-	12.52%	-	-	-	-	-	100.00%	5.63%	1.95%	34.71%
Portfolio 14	25.44%	59.10%	-	-	15.46%	-	-	-	-	-	100.00%	6.07%	2.05%	33.83%
Portfolio 15	31.20%	50.40%	-	-	18.40%	-	-	-	-	-	100.00%	6.61%	2.15%	32.60%
Portfolio 16	36.96%	41.70%	-	-	21.34%	-	-	-	-	-	100.00%	7.22%	2.25%	31.24%
Portfolio 17	42.72%	32.99%	-	-	24.28%	-	-	-	-	-	100.00%	7.88%	2.35%	29.87%
Portfolio 18	48.48%	24.29%	-	-	27.22%	-	-	-	-	-	100.00%	8.59%	2.45%	28.58%
Portfolio 19	54.24%	15.59%	-	-	30.17%	-	-	-	-	-	100.00%	9.33%	2.55%	27.38%
Portfolio 20	60.01%	6.89%	-	-	33.11%	-	-	-	-	-	100.00%	10.10%	2.65%	26.29%
Portfolio 21	77.13%	-	-	-	22.87%	-	-	-	-	-	100.00%	11.05%	2.75%	24.92%
Portfolio 22	**100.00%**	**-**	**-**	**-**	**-**	**-**	**-**	**-**	**-**	**-**	**100.00%**	**12.44%**	**2.79%**	**22.44%**

Combining the data from Table 9 with the risk-return profile illustrated in Figure 4, we can see in Figure 6, a comprehensive understanding of individual stocks, portfolios, and benchmark indexes emerged. Investors, driven by rationality and risk aversion, opt for portfolios with higher yields when faced with similar risk levels (Reilly & Brown, 2006). Conversely, in scenarios with equivalent returns, preference is given to portfolios with lower risk. Consequently, portfolios and stocks positioned beneath the efficient frontier are deemed inefficient.

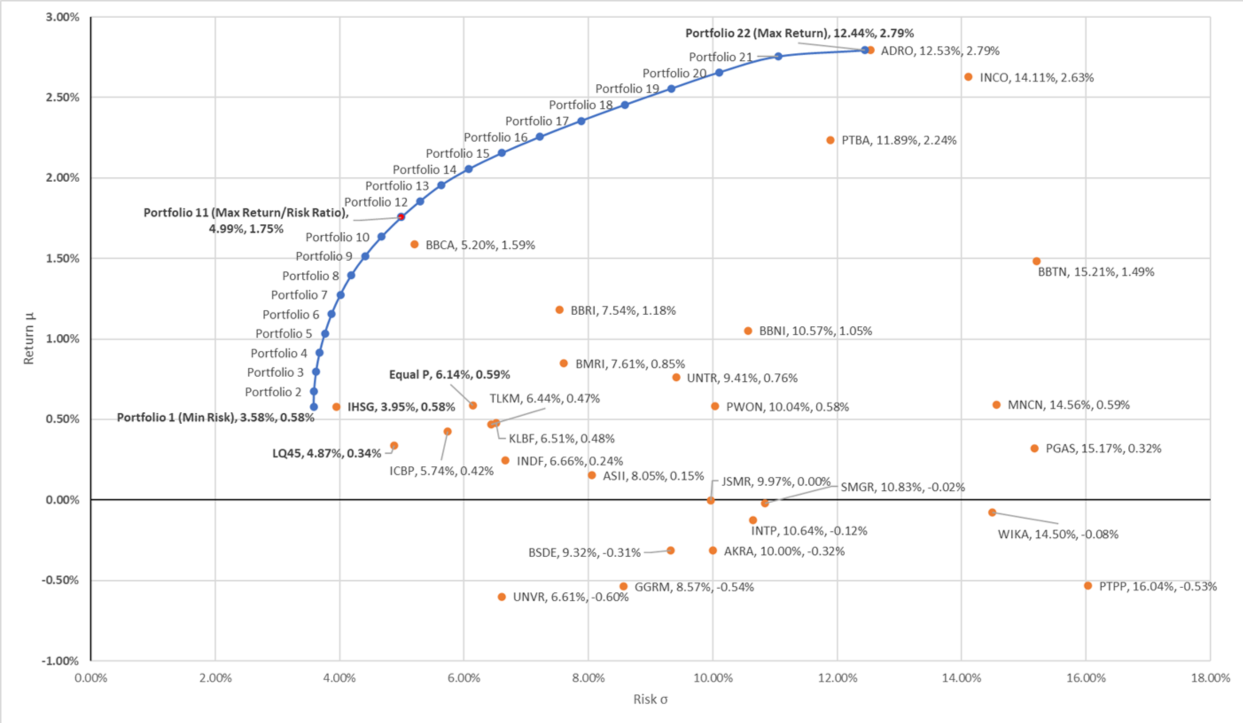


Figure 6 Efficient Frontier

For instance, consider the case of PWON depicted in Figure 6. Investing 100% of funds in PWON yields a risk of 10.04% and a return of 0.58%, which is highly inefficient. Alternatively, diversifying investments across Portfolio 20 offers a higher return of 2.65% at a similar risk level of 10.10%. Similarly, investing in Portfolio 1 mitigates risk to 3.58% while maintaining the same return of 0.58%.

The pinnacle of our analysis, the optimal portfolio derived from the Markowitz model, lies on the efficient frontier curve. Portfolio 11 emerges as the optimal choice, boasting a Sharpe ratio of 35.15%. Comprising 5 stocks with specific weight compositions, including ADRO, BBCA, ICBP, INCO, and PTBA, this portfolio yields an expected return of 1.75% with a risk level of 4.99%.

Data Processing – Single Index Model

The process of data processing in the Single Index Model for analyzing the optimal portfolio is akin to that of the Markowitz model. It involves collecting historical data spanning from 2016 to 2021, selecting instruments, and computing expected returns on both instruments and market index returns, alongside calculating covariance. Additionally, monthly data on the Central Bank of Indonesia's 7-day Reverse Repo Rate (BI7DRR) is incorporated as a proxy for the risk-free return on assets (Table 10). This data aids in evaluating stock return and risk on a monthly basis.

Table 10 Bank Indonesia 7-day Reverse Repo Rate

Period	BI7DRRR (per annum)						BI7DRRR (per month)					
	2016	2017	2018	2019	2020	2021	2016	2017	2018	2019	2020	2021
January	6.00%	4.75%	4.25%	6.00%	5.00%	3.75%	0.50%	0.40%	0.35%	0.50%	0.42%	0.31%
February	5.75%	4.75%	4.25%	6.00%	4.75%	3.50%	0.48%	0.40%	0.35%	0.50%	0.40%	0.29%
March	5.50%	4.75%	4.25%	6.00%	4.50%	3.50%	0.46%	0.40%	0.35%	0.50%	0.38%	0.29%
April	5.50%	4.75%	4.25%	6.00%	4.50%	3.50%	0.46%	0.40%	0.35%	0.50%	0.38%	0.29%
May	5.50%	4.75%	4.63%	6.00%	4.50%	3.50%	0.46%	0.40%	0.39%	0.50%	0.38%	0.29%
June	5.25%	4.75%	5.25%	6.00%	4.25%	3.50%	0.44%	0.40%	0.44%	0.50%	0.35%	0.29%
July	5.25%	4.75%	5.25%	5.75%	4.00%	3.50%	0.44%	0.40%	0.44%	0.48%	0.33%	0.29%
August	5.25%	4.50%	5.50%	5.50%	4.00%	3.50%	0.44%	0.38%	0.46%	0.46%	0.33%	0.29%
September	5.00%	4.25%	5.75%	5.25%	4.00%	3.50%	0.42%	0.35%	0.48%	0.44%	0.33%	0.29%
October	4.75%	4.25%	5.75%	5.00%	4.00%	3.50%	0.40%	0.35%	0.48%	0.42%	0.33%	0.29%
November	4.75%	4.25%	6.00%	5.00%	3.75%	3.50%	0.40%	0.35%	0.50%	0.42%	0.31%	0.29%
December	4.75%	4.25%	6.00%	5.00%	3.75%	3.50%	0.40%	0.35%	0.50%	0.42%	0.31%	0.29%
Average	**4.72%**						**0.39%**					

Source: https://www.bi.go.id/

To compute the Beta (βi) and Alpha (αi) of each stock, monthly returns of individual stocks are compared with IHSG returns using Excel functions like "slope" and "intercept." Average excess return is then determined by deducting the average monthly actual stock return from the corresponding risk-free rate (BI7DRRR) for the same month. This value is divided by the stock's Beta (βi) to derive excess return to beta (ERBi). Stocks with negative ERBi values are subsequently excluded from the optimized portfolio. Variance excess return (σ_{ei}^2) is calculated using Excel's "Var.P" formula with monthly excess return data.

Following this, Ai, Bi, and Ci values are computed for each stock to establish a Cut-Off Point (C*). Ai is derived by dividing the stock's ERBi by its variance excess return (σ_{ei}^2), while Bi is obtained by dividing the stock's squared Beta (βi) by its variance excess return (σ_{ei}^2). Ci is calculated by multiplying Ai by the Market Variance (σ_m^2) divided by one plus Bi multiplied by the Market Variance (σ_m^2). The highest Ci value serves as the Cut-Off Point (C*), which is found to be 0.43%.

Finally, to ascertain stock inclusion in the optimal portfolio, positive Zi values are crucial. These values consider the stock's Beta (βi), variance excess return (σ_{ei}^2), excess return to beta (ERBi), and the Cut-Off Point (C*). Once the stocks are selected, the proportion of funds allocated is based on each stock's Zi value relative to the total Zi value.

Table 10 Calculation Process – Single Index Model

Stock	Return	Beta	Alpha	Excess Return	Excess Return to Beta	Variance of Excess Return	Filter 1	Ai	Bi	Ci	C*	Zi	Filter 2	Wi
ADRO	2.79%	1.34	2.01%	2.40%	1.79%	1.57%	Passed	2.05	114.91	0.27%	0.43%	0.52	Passed	17.71%
AKRA	-0.32%	1.74	-1.33%	-0.71%	-0.41%	1.00%	Eliminated	-	-	-	-	-	-	-
ASII	0.15%	1.32	-0.61%	-0.24%	-0.18%	0.65%	Eliminated	-	-	-	-	-	-	-
BBCA	1.59%	0.94	1.04%	1.20%	1.27%	0.27%	Passed	4.17	328.46	0.43%	0.43%	1.75	Passed	59.54%
BBNI	1.05%	2.05	-0.14%	0.66%	0.32%	1.12%	Passed	1.21	375.10	0.12%	0.43%	(0.85)	Eliminated	-
BBRI	1.18%	1.45	0.34%	0.79%	0.54%	0.57%	Passed	2.01	369.28	0.20%	0.43%	(0.83)	Eliminated	-
BBTN	1.49%	2.49	0.04%	1.09%	0.44%	2.31%	Passed	1.17	267.05	0.13%	0.43%	(0.44)	Eliminated	-
BMRI	0.85%	1.39	0.04%	0.46%	0.33%	0.58%	Passed	1.10	334.02	0.11%	0.43%	(1.14)	Eliminated	-
BSDE	-0.31%	1.64	-1.26%	-0.70%	-0.43%	0.87%	Eliminated	-	-	-	-	-	-	-
GGRM	-0.54%	0.92	-1.07%	-0.93%	-1.01%	0.73%	Eliminated	-	-	-	-	-	-	-
ICBP	0.42%	0.30	0.25%	0.03%	0.10%	0.33%	Passed	0.03	27.99	0.00%	0.43%	(1.27)	Eliminated	-
INCO	2.63%	1.63	1.68%	2.23%	1.37%	1.99%	Passed	1.83	132.98	0.24%	0.43%	0.14	Passed	4.85%
INDF	0.24%	0.60	-0.11%	-0.15%	-0.24%	0.44%	Eliminated	-	-	-	-	-	-	-
INTP	-0.12%	1.34	-0.90%	-0.52%	-0.39%	1.13%	Eliminated	-	-	-	-	-	-	-
JSMR	0.00%	1.49	-0.87%	-0.40%	-0.27%	0.99%	Eliminated	-	-	-	-	-	-	-
KLBF	0.48%	0.74	0.04%	0.08%	0.11%	0.42%	Passed	0.15	130.64	0.02%	0.43%	(1.41)	Eliminated	-
MNCN	0.59%	1.88	-0.50%	0.20%	0.11%	2.12%	Passed	0.18	167.07	0.02%	0.43%	(0.52)	Eliminated	-
PGAS	0.32%	2.54	-1.16%	-0.07%	-0.03%	2.30%	Eliminated	-	-	-	-	-	-	-
PTBA	2.24%	1.14	1.58%	1.84%	1.62%	1.41%	Passed	1.48	91.41	0.20%	0.43%	0.52	Passed	17.89%
PTPP	-0.53%	3.05	-2.30%	-0.92%	-0.30%	2.57%	Eliminated	-	-	-	-	-	-	-
PWON	0.58%	1.66	-0.38%	0.19%	0.12%	1.01%	Passed	0.31	273.03	0.03%	0.43%	(0.98)	Eliminated	-
SMGR	-0.02%	1.56	-0.93%	-0.41%	-0.27%	1.17%	Eliminated	-	-	-	-	-	-	-
TLKM	0.47%	0.79	0.01%	0.08%	0.10%	0.41%	Passed	0.15	149.74	0.02%	0.43%	(1.52)	Eliminated	-
UNTR	0.76%	0.76	0.32%	0.37%	0.49%	0.89%	Passed	0.31	64.42	0.04%	0.43%	(0.26)	Eliminated	-
UNVR	-0.60%	0.51	-0.89%	-0.99%	-1.96%	0.43%	Eliminated	-	-	-	-	-	-	-
WIKA	-0.08%	2.41	-1.48%	-0.47%	-0.20%	2.10%	Eliminated	-	-	-	-	-	-	-

Based on calculation presented in Table 10, the optimal portfolio from the Single Index model in this research consisted of 4 stocks, with weight composition: 17.71% ADRO, 59.54% BBCA, 4.85% INCO, 17.89% PTBA. This portfolio produced 1.97% of expected monthly return with 1.08 beta (Table 11).

Table 11 Optimal Portfolio – Single Index Model

Stock	Return	Beta	Wi	Portfolio Return	Portfolio Beta
ADRO	2.79%	1.34	17.71%	0.49%	0.24
BBCA	1.59%	0.94	59.54%	0.95%	0.56
INCO	2.63%	1.63	4.85%	0.13%	0.08
PTBA	2.24%	1.14	17.89%	0.40%	0.20
Portfolio				**1.97%**	**1.08**

Portfolio Performance Review and Hypotheses Testing

Now that we have 2 optimized portfolios from each model, we will assess their performance by conducting investment simulations. The collected sample data spans from January 2016 to December 2021. Following the fund allocation from both models, as shown in Figure 7, we simulate investing our fund on January 1, 2022, and hold it until December 31, 2023. We will compare how both portfolios perform in terms of average return, risk (standard deviation), Sharpe ratio, and Treynor ratio against IHSG, LQ45 index, and each other.

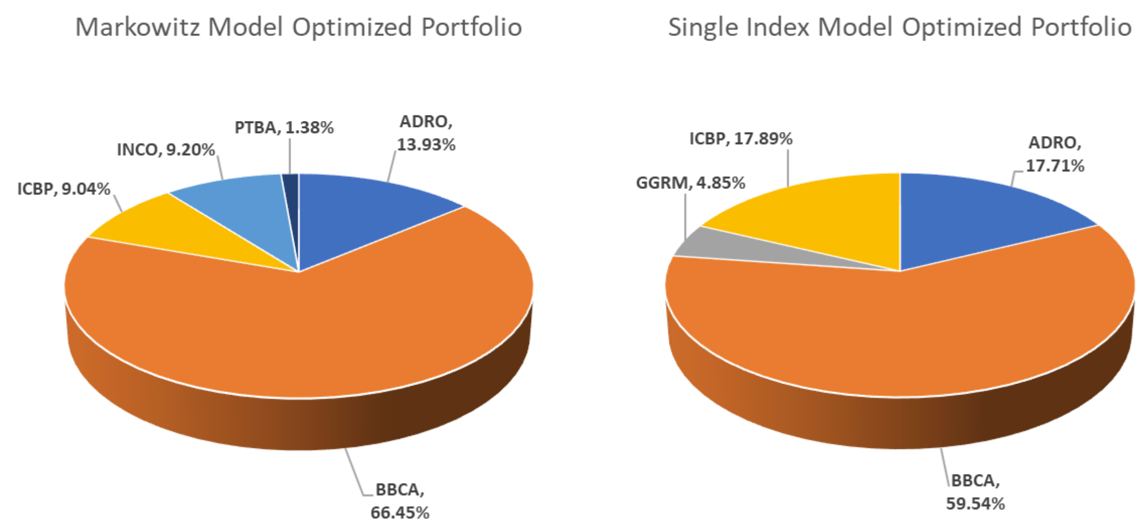


Figure 7 Markowitz & Single Index Model Optimized Portfolio

- Markowitz Optimized Portfolio vs IHSG and LQ45
  Based on calculations presented in Table 12, the Markowitz portfolio outperforms the market (IHSG) and LQ45 index in terms of yearly return, monthly average return, Sharpe ratio, and Treynor ratio in 2022. However, in 2023, it underperformed both IHSG and LQ45, yielding negative returns. Thus, the level of risk and return obtained in Markowitz's optimal portfolio model cannot consistently outperformed the market and the index benchmark. H01 is rejected, Ha1 is accepted.

Table 12 Portfolio Performance Evaluation

	Remark	LQ 45	IHSG	Markowitz	Single Index
2022	Yearly Return	0.61%	4.06%	27.39%	31.06%
	Average Monthly Return	0.13%	0.36%	2.26%	2.53%
	Risk (Monthly Stdev)	4.20%	2.42%	4.74%	5.41%
	Beta	1.64	1	1.59	1.90
	Sharpe Ratio	-4.69%	1.30%	40.76%	40.57%
	Treynor Ratio	-0.12%	0.03%	1.22%	1.16%
2023	Yearly Return	3.57%	6.17%	-3.70%	-11.23%
	Average Monthly Return	0.33%	0.53%	-0.19%	-0.71%
	Risk (Monthly Stdev)	2.87%	2.55%	4.74%	5.41%
	Beta	0.92	1	0.87	1.28
	Sharpe Ratio	0.01%	7.82%	-10.88%	-19.24%
	Treynor Ratio	0.00%	0.20%	-0.59%	-0.82%

- Single Index Optimized Portfolio vs IHSG and LQ45
  Similarly, the Single Index portfolio outperforms the market (IHSG) and LQ45 index in terms of yearly return, monthly average return, Sharpe ratio, and Treynor ratio in 2022, as indicated in Table 12. Nevertheless, the portfolio demonstrated negative returns in 2023 and it underperformed both IHSG and LQ45 in all the metrics. Thus, the level of risk and return obtained in Single Index optimal portfolio model cannot consistently outperformed the market and the index benchmark. H02 is rejected, Ha2 is accepted.
- Markowitz Optimized Portfolio vs Single Index Optimized Portfolio
  The performance of the Markowitz and Single Index Optimized Portfolios in 2022 and 2023 reveals notable trends. In 2022, both portfolios yielded strong returns, with the Single Index Portfolio slightly edging out the Markowitz Portfolio in yearly and average monthly returns. Despite this, the Markowitz Portfolio exhibited lower risk levels. Conversely, in 2023, both portfolios faced challenges, resulting in negative returns. The Markowitz Portfolio experienced a smaller decline compared to the Single Index Portfolio. Overall, while the Single Index Portfolio showed slightly higher returns in 2022, the Markowitz Portfolio maintained lower risk levels. In 2023, despite negative returns for both portfolios, the Markowitz Portfolio demonstrated a more favorable performance in terms of risk-adjusted metrics. These findings suggest that the Markowitz Portfolio may offer a more stable and consistent investment option over the analyzed period, with H03 accepted and Ha3 rejected.

Conclusion and Recommendations

The research findings underscore the nuanced performance dynamics of portfolio optimization models, particularly the Markowitz and Single Index models, in the context of Indonesian stocks. Analysis over the years 2022 and 2023 reveals distinct trends in portfolio performance. In 2022, both the Markowitz and Single Index portfolios demonstrated strong returns, outperforming the market (IHSG) and the LQ45 index across various metrics. However, in 2023, both portfolios experienced challenges, resulting in negative returns. While the Single Index Portfolio showed marginally higher returns in 2022, the Markowitz Portfolio maintained lower risk levels. Conversely, in 2023, the Markowitz Portfolio exhibited a more favorable performance in terms of risk-adjusted metrics despite negative returns for both portfolios. These findings suggest that the Markowitz Portfolio may offer a more stable and consistent investment option over the analyzed period.

The research outcomes offer valuable insights for portfolio managers and investors seeking to optimize their investment strategies. Despite the challenges faced in 2023, both the Markowitz and Single Index models exhibited strengths in 2022, showcasing their potential for generating strong returns. Investors can leverage these models to construct diversified portfolios tailored to their risk preferences and return objectives. The Markowitz Portfolio, in particular, may offer a more resilient investment option, maintaining lower risk levels over the analyzed period. Portfolio managers can utilize these insights to refine their asset allocation strategies and optimize risk-return profiles for their portfolios.

While this research provides valuable insights, several limitations should be acknowledged for future studies. The analysis focuses exclusively on the Indonesian stock market, using the Composite Index (IHSG) and LQ 45 Index. Future research could explore other stock exchanges and indexes to validate the findings across diverse market conditions. Additionally, incorporating daily stock prices and alternative portfolio construction periods could enhance the accuracy and robustness of the analysis. Furthermore, investigating market timing theories and their impact on portfolio performance could provide additional depth to the research and offer valuable insights for practitioners navigating financial markets.

By addressing these limitations and pursuing further research avenues, scholars can advance our understanding of portfolio optimization techniques and provide practitioners with enhanced tools and strategies for achieving their investment objectives in dynamic market environments.

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